Discrete-Time IDA Passivity-Based Control of Coupled Tank Processes Subject to Actuator Saturation

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Outline

- What is IDA-PBC, why is it useful & what are limitations?
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What is IDA-PBC, why is it useful & what are limitations?

- Interconnection damping assignment passivity-based control (IDA-PBC)
  - Ortega et. all 2002, Ramirez et. all 2009, **Johnsen & Allgower 2007**.
- Why is it useful?
  - Allows for **low-complexity** automated controller design for certain classes of underactuated and nonminimum-phase systems.
  - Results in explicit **gain limits** which can be **enforced at run-time** in order to restrict the operator interface to prevent entry of potentially destabilizing controller gain values.
  - Can **explicitly determine feasible set-point constraints** to in order to avoid potentially uncontrollable operating conditions.
  - Allows for integrators to be introduced in order to compensate for model uncertainty and actuator degradation.
- Limitations
  - **Anti-windup control needs further study**, we present a preliminary solution which relies on a linear systems assumptions; however, our solution works reasonably well for control of coupled-tank processes.
  - **Theory is based on continuous-time controller assumptions**, we used **bilinear-transform** to approximate integrators in order to achieve reasonably low sampling rates.
  - **Assertions** were made by Johnsen & Allgower that techniques could be applied to **nonminimum phase systems**; however, their **controller constraints** for a nonminimum four tank system **were incorrect**.

Problem Setup: Dynamics

Use IDA-PBC for nonlinear dynamics systems with input-affine dynamics and extended with integrator states as shown here

$$\dot{x} = f(x) + g(x)u$$

$$\dot{x}_I = k_I(f_I(x_c) - f_I(x_c^*))$$

$$\ldots = \begin{bmatrix} k_{I1}(f_{I1}(x_{c1}) - f_{I1}(x_{c1}^*)) & & & 0 \\ & k_{I2}(f_{I2}(x_{c2}) - f_{I2}(x_{c2}^*)) & & \\ & & \ddots & \vdots \\ 0 & & \cdots & k_{Ip}(f_{Ip}(x_{cp}) - f_{Ip}(x_{cp}^*)) \end{bmatrix}$$

Variable	Domain	Description
x	$\mathbb{R}^n$	System state trajectory
x^*	$\mathbb{R}^n$	Desired state trajectory
x_c^*	$\mathbb{R}^p$	Controllable subset of desired states
x_I	$\mathbb{R}^p$	Integrator states
u	$\mathbb{R}^m$	Feedback control input $u = \beta(x, x_I, x^*)$

Problem Setup: Constraints

$$\begin{bmatrix} \dot{x} \\ \dot{x}_I \end{bmatrix} = Q_d(x, x_I)\nabla H_d(x, x_I, x^*)$$

Final control law will have this form, where H_d is the final Hamiltonian.

$$Q_d(x, x_I) < 0$$

$$\nabla H_d(x = x^*, x_I = 0, x^*) = 0$$

$$\nabla^2 H_d(x = x^*, x_I = 0, x^*) > 0$$

The goal is to find β to get the form shown above, and satisfy the constraints. This implies that at steady state, the following two conditions hold:

$$x = x^*$$

$$x_I = 0$$

Control: IDA-PBC Procedure

1. Augment the state-space description with p additional integrators

$$\begin{bmatrix} \dot{x} \\ \dot{x}_I \end{bmatrix} = \begin{bmatrix} f(x) \\ k_I(f_I(x_c) - f_I(x_c^*)) \end{bmatrix} + \begin{bmatrix} g(x) \\ 0 \end{bmatrix} u$$

2. Choose a candidate Hamiltonian H which depends on additional scaling terms $k_i, i \in \{1, \ldots, n\}$ such that

$$\begin{bmatrix} \dot{x} \\ \dot{x}_I \end{bmatrix} = Q_d \nabla H(x, x_I) = \begin{bmatrix} f(x) \\ k_I(f_I(x_c) - f_I(x_c^*)) \end{bmatrix}$$

$Q_d \in \mathbf{R}^{(n+p)\times(n+p)}$ is a constant matrix.

Control: IDA-PBC Procedure

3. Find conditions on the coefficients k_i and k_I to render Q_d negative definite. There are multiple ways to satisfy this. We apply Sylvester's criterion on the negative Hermitian of Q_d:

$$-\mathrm{He}\{Q_d\} = -\frac{1}{2}(Q_d^{\mathrm{T}} + Q_d) > 0$$

4. Find a matrix P satisfying the following conditions:

i) $g^{\perp} Q_d P = 0$ and ii) $g^{\dagger} Q_d P = -I$ in which $P \in \mathbf{R}^{(n+p)\times m}$

$g^{\perp} \in \mathbf{R}^{(n-m)\times(n+p)}$ is the max rank left annihilator of $g(x)$ and

$g^{\dagger} \in \mathbf{R}^{m\times(n+p)}$ is the left inverse of $g(x)$ such that:

$g^{\perp}\left[g^{\mathrm{T}}, 0^{\mathrm{T}}\right]^{\mathrm{T}} = 0$ and $g^{\dagger}\left[g^{\mathrm{T}}, 0^{\mathrm{T}}\right]^{\mathrm{T}} = I$ respectively.

Control: IDA-PBC Procedure

5. Solve for the desired Hamiltonian H_d and pos. def. symmetric matrix Q using the following form:

$$H_d(x, x_I, x^*) = H(x, x_I) + \frac{1}{2}\tilde{z}^{\mathrm{T}} Q \tilde{z} + l^{\mathrm{T}} z$$

$$z = P^{\mathrm{T}} \begin{bmatrix} x \\ x_I \end{bmatrix}, \tilde{z} = P^{\mathrm{T}} \tilde{x} = P^{\mathrm{T}} \begin{bmatrix} (x - x^*) \\ (x_I - 0) \end{bmatrix}$$

We use the gradient and Hessian constraints to solve for l, and to find expressions for the uncontrollable parts of x^*.

$$\nabla H_d(x, x_I, x^*) = \nabla H(x, x_I) + PQP^{\mathrm{T}} \tilde{x} + Pl$$

$$\nabla H_d(x = x^*, x_I = 0, x^*) = \nabla H(x = x^*, x_I = 0) + Pl = 0$$

$$\nabla^2 H_d(x = x^*, x_I = 0, x^*) = \nabla^2 H(x = x^*, x_I = 0) + PQP^{\mathrm{T}} > 0$$

Control: IDA-PBC Procedure

6. Determine the control law:

$$\beta(x) = g^{\dagger}\left\{Q_d \nabla H_d(x, x_I, x^*) - \begin{bmatrix} f(x) \\ k_I(f_I(x_c) - f_I(x_c^*)) \end{bmatrix}\right\}$$

$$\beta(x, x_I, x^*) = g^{\dagger}\left\{Q_d P(QP^T \tilde{x} + l)\right\}$$

Because P was normalized such that $g^{\dagger}Q_d P = -I$, the control law can be simplified down to the following form:

$$\beta(x, x_I, x^*) = -K_p \tilde{x} + u^*$$

$$K_p = QP^T, u^* = -l$$

Control: Integrator Anti-Windup Compensator

$$\dot{x}_1 = k_I(f_I(x_{\text{linc}}) - f_I(x_c^*)),\ x_{\text{linc}} = \bar{x}_c + x_c$$

$$\dot{\bar{x}} = A(x^*)\bar{x} + g\bar{u}$$

$$\bar{u} = u - \text{sat}(u, u_{\min}, u_{\max})$$

$$u = \beta(x, x_I, x^*)$$

$$A(x) = \begin{bmatrix} \frac{\partial f_1(x)}{\partial x_1} & \cdots & \frac{\partial f_1(x)}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n(x)}{\partial x_1} & \cdots & \frac{\partial f_n(x)}{\partial x_n} \end{bmatrix}$$

$$\text{sat}(u, u_{\min}, u_{\max}) = \begin{cases} u & \text{if} \quad u_{\min} \le u \le u_{\max} \\ u_{\min} & \text{if} \quad u \le u_{\min} \\ u_{\max} & \quad \text{otherwise.} \end{cases}$$

Control: DT-Implementation

Discrete-time implementation of the full controller with anti-windup:

$$u(k) = \beta(x(kT_s), x_I(k-1), x^*(k))$$

$$\bar{u}(k) = u(k) - \mathrm{sat}(u(k), u_{\min}, u_{\max})$$

$$\dot{\bar{x}}(k) = A(x^*(k))\bar{x}(k) + g\bar{u}(k)$$

$$\bar{x}(k) = \bar{x}(k-1) + \frac{T_s}{2}\left[\dot{\bar{x}}(k) + \dot{\bar{x}}(k-1)\right]$$

$$x_{\mathrm{linc}}(k) = \bar{x}_c(k) + x_c(kT_s) \in [x_{\min}, x_{\max}]$$

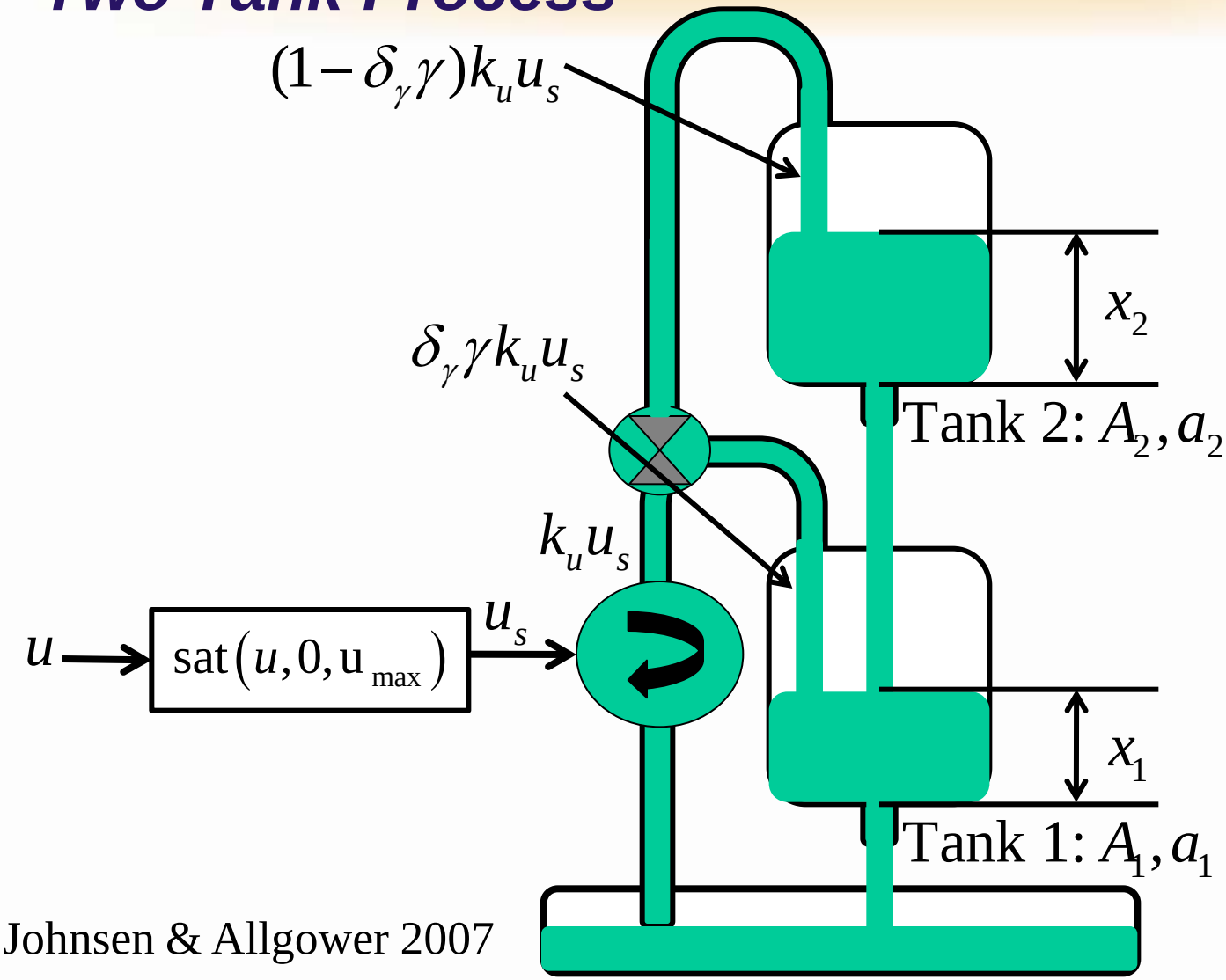
$$\dot{x}_I(k) = k_I[f_I(x_{\mathrm{linc}}(k)) - f_I(x_c^*(k))]$$

$$x_I(k) = x_I(k-1) + \frac{T_s}{2}[\dot{x}_I(k) + \dot{x}_I(k-1)]$$

Symbolic Analysis using MuPAD

1. Create a matrix Q_d with symbolic variable entries. Create an expression for the determinant of each leading principal submatrix $|-He\{Q_d\}_k|$ for $k=1\ldots n+p$. Use *solve()* to find the control coefficient bounds subject to the constraints $|-He\{Q_d\}_k|>0$.
2. Finding the matrix P requires several steps:
   1. Use *linalg::nullspace()* with $g(x)^T$, then concatenate the resulting vectors to find $g^{\perp}(x)$.
   2. Compute $g\dagger$ using the Moore-Penrose left pseudoinverse $g\dagger(x)=(g(x)^T g(x))^{-1} g(x)^T$.
   3. Form $g^{\perp}(x)Q_d$ and $g\dagger(x)Q_d$. Use *linalg::nullspace()* to get P_{null} (as above), and then $P=-P_{\text{null}}(g\dagger(x)Q_d P_{\text{null}})^{-1}$.

Two Tank Process



Johnsen & Allgower 2007

Two Tank Process

Two tank dynamics

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{-a_1\sqrt{2gx_1} + a_2\sqrt{2gx_2}}{A_1} \\ -\dfrac{a_2\sqrt{2gx_2}}{A_2} \end{bmatrix} + \begin{bmatrix} \dfrac{\delta_\gamma \gamma}{A_1} \\ \dfrac{1-\delta_\gamma \gamma}{A_2} \end{bmatrix} k_u u$$

$$\dot{x}_{I1} = a_1\sqrt{2g}\,(\sqrt{x_1^*} - \sqrt{x_1})$$

Hamiltonian

$$H(x) = \sum_{i=1}^{2} \frac{2}{3} k_i a_i \sqrt{2g}\, x_i^{\frac{3}{2}} + k_{I1} a_1 \sqrt{x_1^*}\, x_{I1}$$

Two Tank Process

Constant matrix

$$Q_d = \begin{bmatrix} -\frac{1}{A_1 k_1} & \frac{1}{A_1 k_2} & 0 \\ 0 & -\frac{1}{A_2 k_2} & 0 \\ \frac{k_{I1}}{k_1} & 0 & -\frac{k_{I1}}{k_1} \end{bmatrix}$$

Coefficient bounds

$$0 < k_1 < \frac{(4 - A_1 k_{I1}) A_1 k_2}{A_2}$$

$$0 < k_2 < \infty$$

$$0 < k_{I1} < \frac{4}{A_1}$$

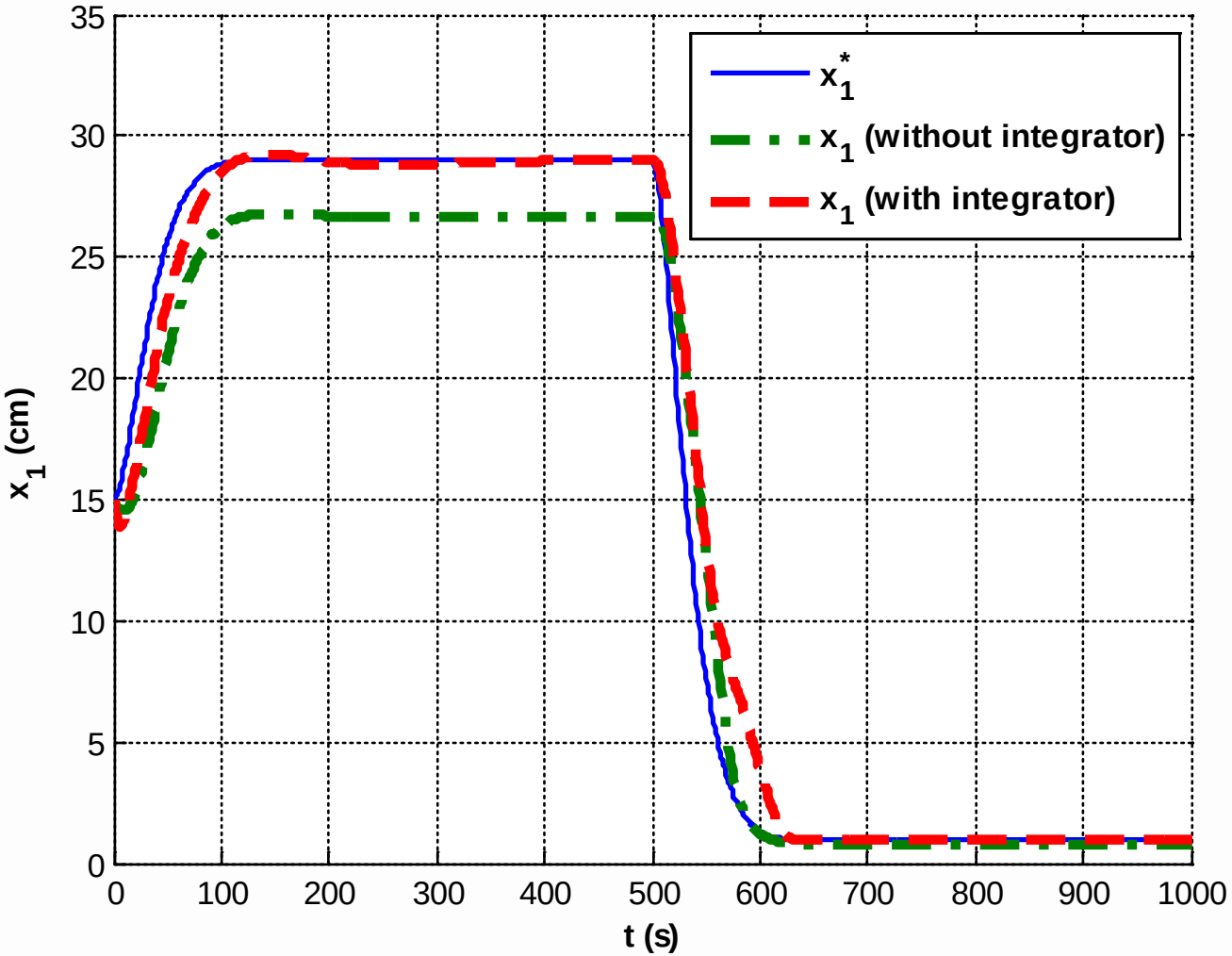
Control terms

$$P = \begin{bmatrix} k_1 \\ k_2(1-\gamma) \\ k_1 \end{bmatrix}$$

$$l = -a_1 \sqrt{2 g x_1}$$

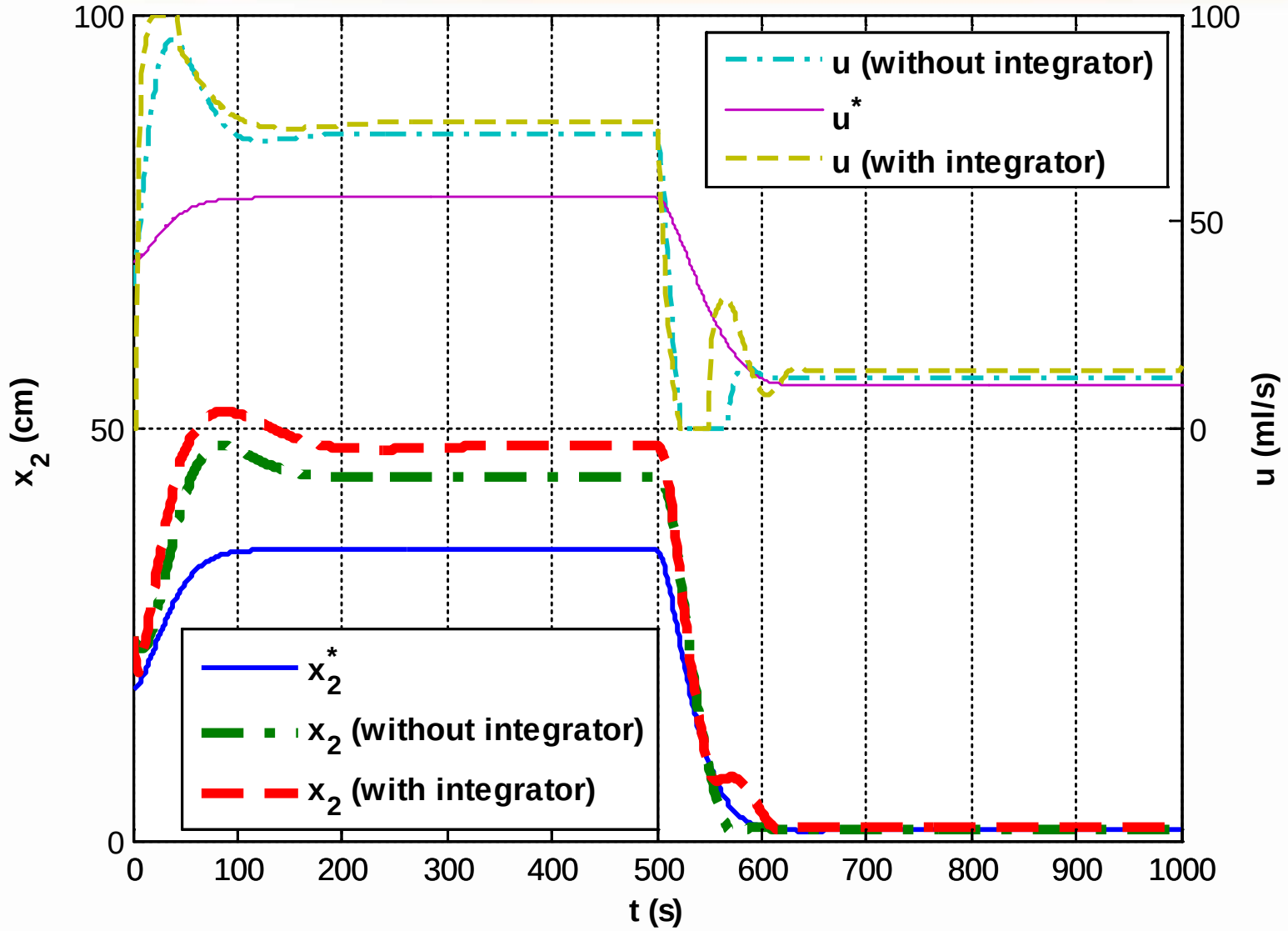
$$x_2^* = \frac{a_1^2 (1-\gamma)^2}{a_2^2} x_1^*$$

Two Tank Process Step Response

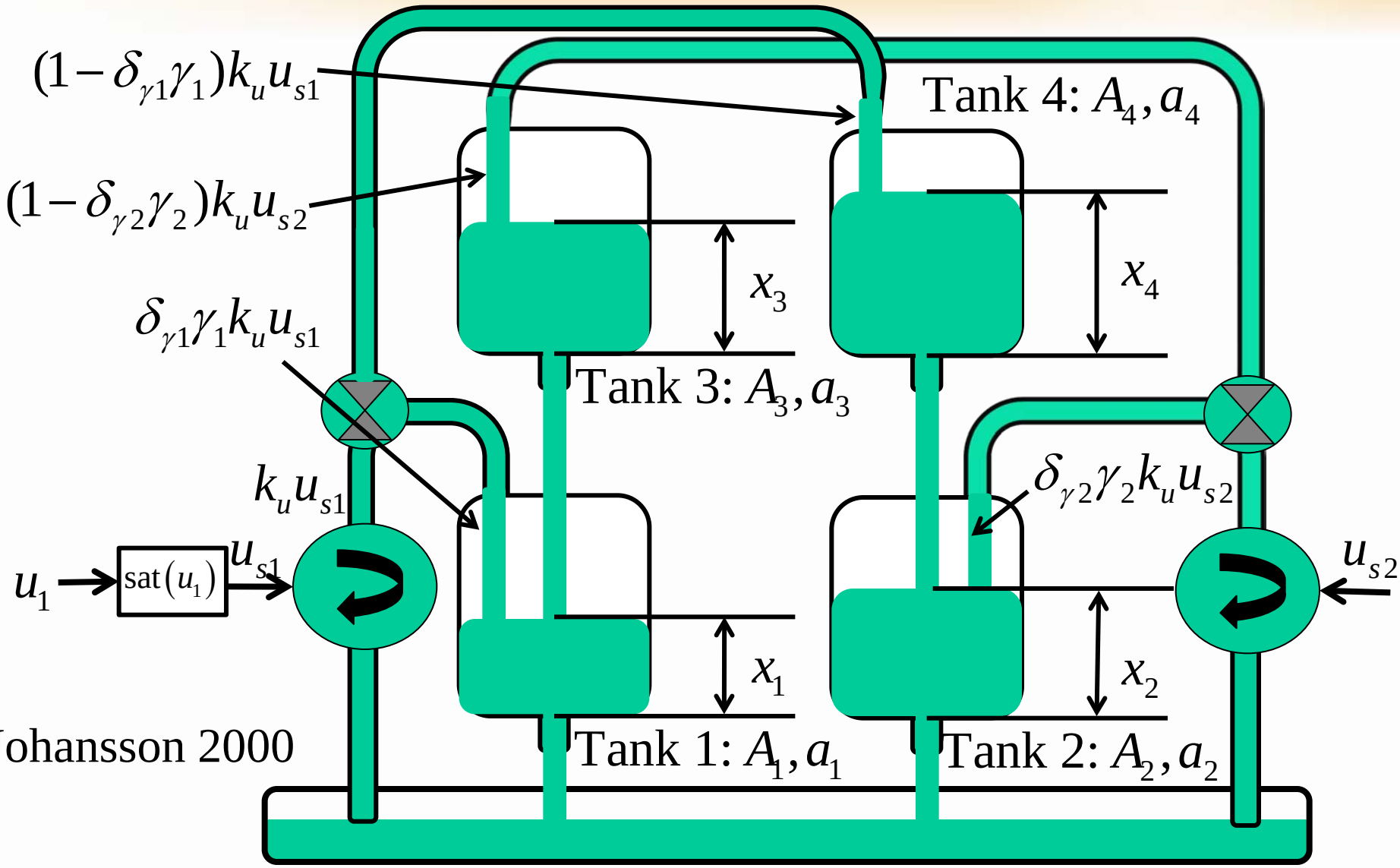


$$\delta_\gamma = k_u = .75$$

Two Tank Process Step Response



Four Tank Process



Johansson 2000

Four Tank Process

Dynamics

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \dfrac{-a_1\sqrt{2gx_1} + a_3\sqrt{2gx_3}}{A_1} \\ \dfrac{-a_2\sqrt{2gx_2} + a_4\sqrt{2gx_4}}{A_2} \\ -\dfrac{a_3\sqrt{2gx_3}}{A_3} \\ -\dfrac{a_4\sqrt{2gx_4}}{A_4} \end{bmatrix} + \begin{bmatrix} \dfrac{\delta_{\gamma 1}\gamma_1}{A_1} & 0 \\ 0 & \dfrac{\delta_{\gamma 2}\gamma_2}{A_2} \\ 0 & \dfrac{1-\delta_{\gamma 2}\gamma_2}{A_3} \\ \dfrac{1-\delta_{\gamma 1}\gamma_1}{A_4} & 0 \end{bmatrix} k_u \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_{I1} \\ \dot{x}_{I2} \end{bmatrix} = \begin{bmatrix} k_{I1}a_1\sqrt{2g}(\sqrt{x_1^*} - \sqrt{x_1}) \\ k_{I2}a_2\sqrt{2g}(\sqrt{x_2^*} - \sqrt{x_2}) \end{bmatrix}$$

Four Tank Process

Hamiltonian

$$H(x) = \sum_{i=1}^{4} \frac{2}{3} k_i a_i \sqrt{2g}\, x_i^{\frac{3}{2}} + \sum_{j=1}^{2} k_{Ij} a_j \sqrt{x_j^*}\, x_{Ij}$$

Constant matrix

$$Q_d = \begin{bmatrix} \frac{-1}{A_1 k_1} & 0 & \frac{1}{A_3 k_3} & 0 & 0 & 0 \\ 0 & \frac{-1}{A_2 k_2} & 0 & \frac{1}{A_2 k_4} & 0 & 0 \\ 0 & 0 & \frac{-1}{A_3 k_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{A_4 k_4} & 0 & 0 \\ \frac{k_{I1}}{k_1} & 0 & 0 & 0 & -\frac{k_{I1}}{k_1} & 0 \\ 0 & \frac{k_{I2}}{k_2} & 0 & 0 & 0 & -\frac{k_{I2}}{k_2} \end{bmatrix}$$

Johnsen & Allgower 2007

Four Tank Process

Coefficient bounds

$$0 < k_3, k_4 < \infty$$

$$0 < k_1 < \frac{(4 - k_{I1}A_1)k_3}{A_3} \qquad 0 < k_{I1} < \frac{4}{A_1}$$

$$0 < k_2 < \frac{(4 - k_{I2}A_2)k_4}{A_4} \qquad 0 < k_{I2} < \frac{4}{A_2}$$

Control terms

$$P = \begin{bmatrix} \gamma_1 k_1 & (\gamma_2 - 1)k_1 \\ (1-\gamma_1)k_2 & \gamma_2 k_2 \\ 0 & (1-\gamma_2)k_3 \\ (1-\gamma_1)k_4 & 0 \\ \gamma_1 k_1 & (1-\gamma_2)k_1 \\ (1-\gamma_1)k_2 & \gamma_2 k_2 \end{bmatrix}$$

$$\begin{bmatrix} l_1 \\ l_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{\gamma_2 a_1 \sqrt{2gx_1^*} + (\gamma_2 - 1)a_2\sqrt{2gx_2^*}}{\gamma_1 + \gamma_2 - 1} \\ -\dfrac{(\gamma_1 - 1)a_1\sqrt{2gx_1^*} + \gamma_1 a_2 \sqrt{2gx_2^*}}{\gamma_1 + \gamma_2 - 1} \end{bmatrix}$$

$$\begin{bmatrix} x_3^* \\ x_4^* \end{bmatrix} = \begin{bmatrix} \dfrac{(1-\gamma_2)^2}{2ga_3^2} l_2^2 \\ \dfrac{(1-\gamma_1)^2}{2ga_4^2} l_1^2 \end{bmatrix}$$

Four Tank Process

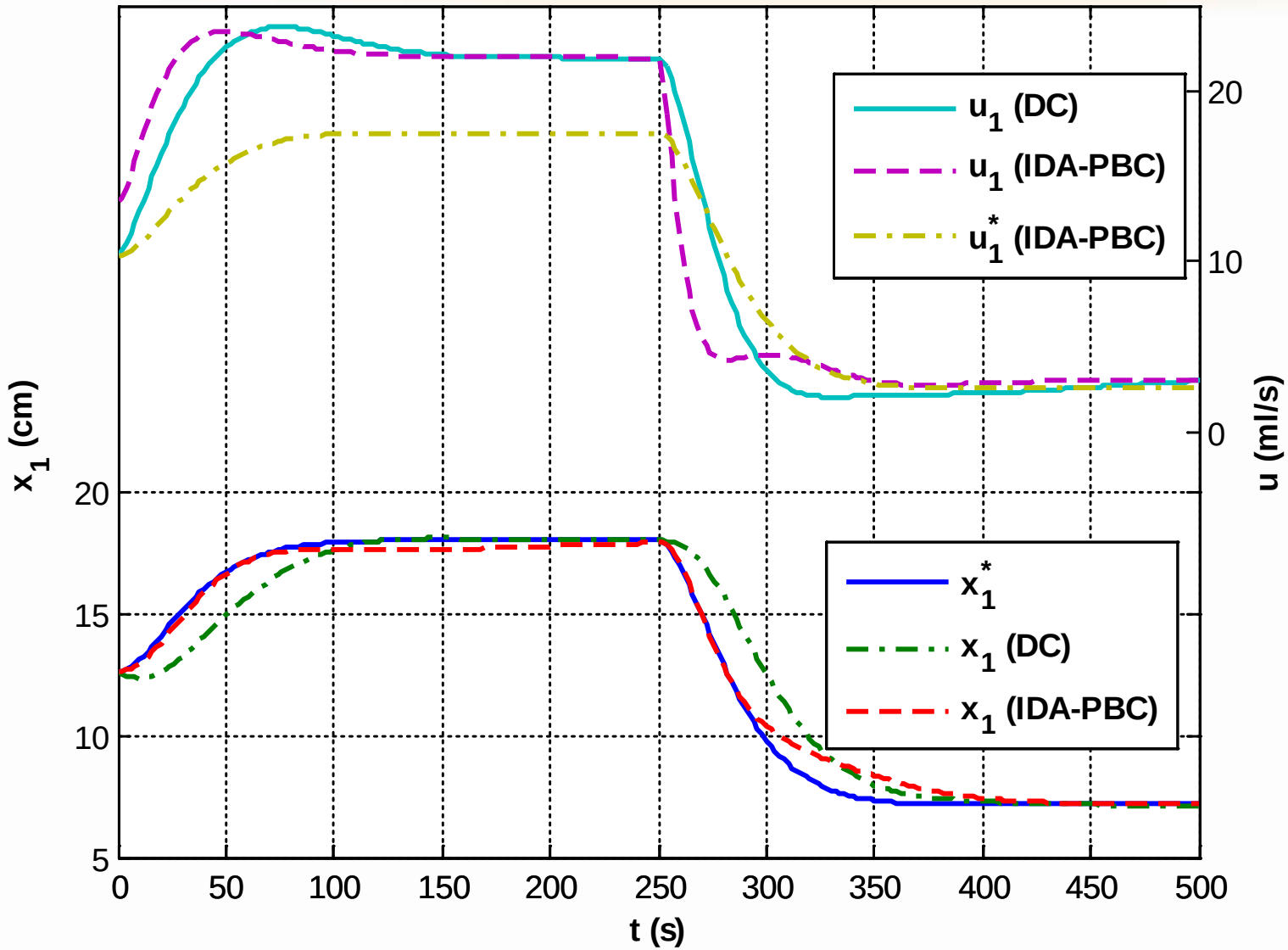
Set point constraints for minimum-phase system

$$\left(\frac{a_1\gamma_2}{a_2(1-\gamma_2)}\right)^2 < \frac{x_2^*}{x_1^*} < \left(\frac{a_1(1-\gamma_1)}{a_2\gamma_1}\right)^2 \quad \text{if} \quad (\gamma_1+\gamma_2)>1$$

Set point constraints for nonminimum-phase system

$$\left(\frac{a_1(1-\gamma_1)}{a_2\gamma_1}\right)^2 < \frac{x_2^*}{x_1^*} < \left(\frac{a_1\gamma_2}{a_2(1-\gamma_2)}\right)^2 \quad \text{if} \quad (\gamma_1+\gamma_2)>1$$

Four Tank Process Step Response (min-phase)



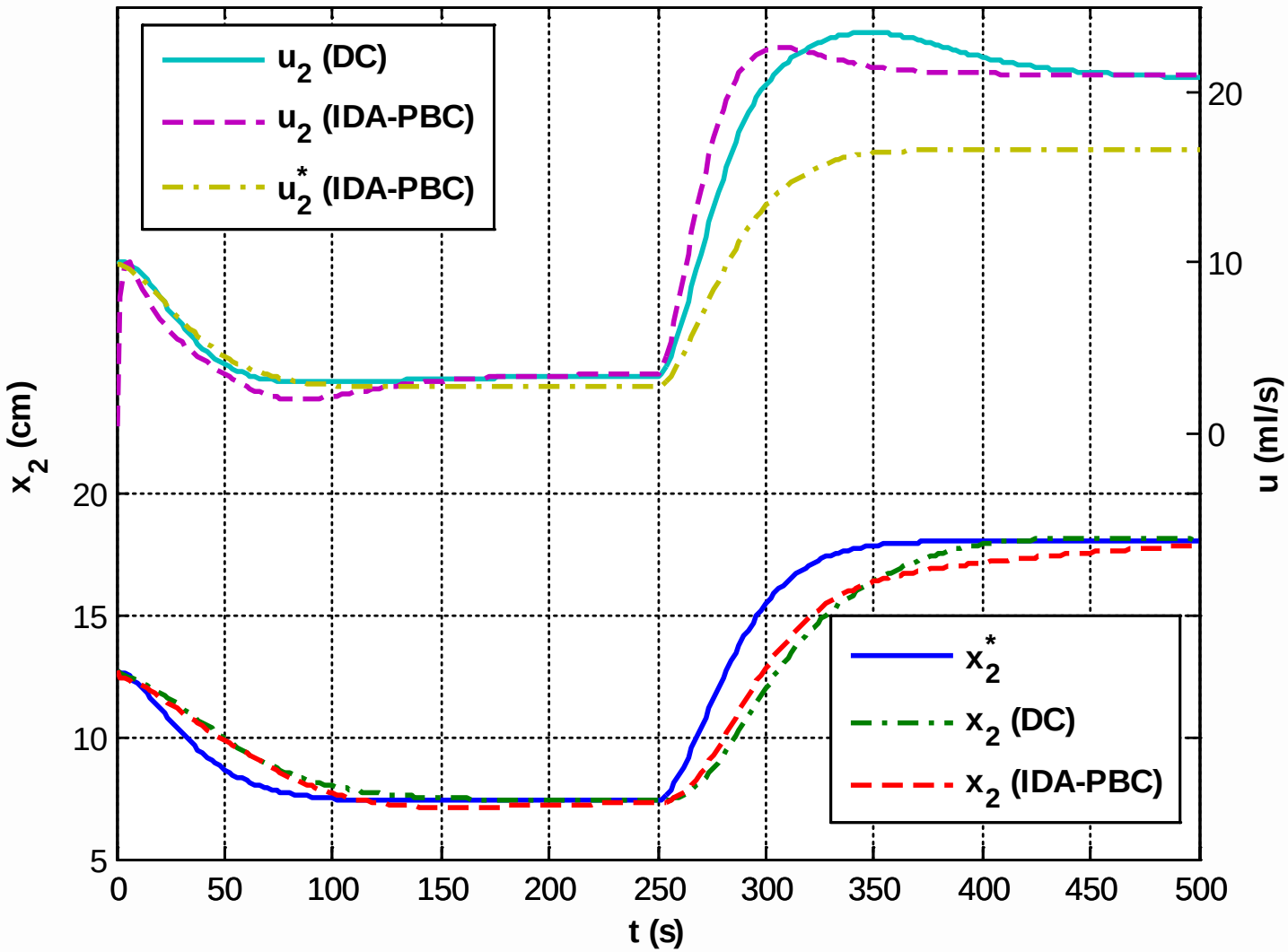
$$\gamma_1 = 0.7$$

$$\gamma_2 = 0.6$$

$$k_u = 0.8$$

$$T_s = 0.1\ \mathrm{s}$$

Four Tank Process Step Response (min-phase)



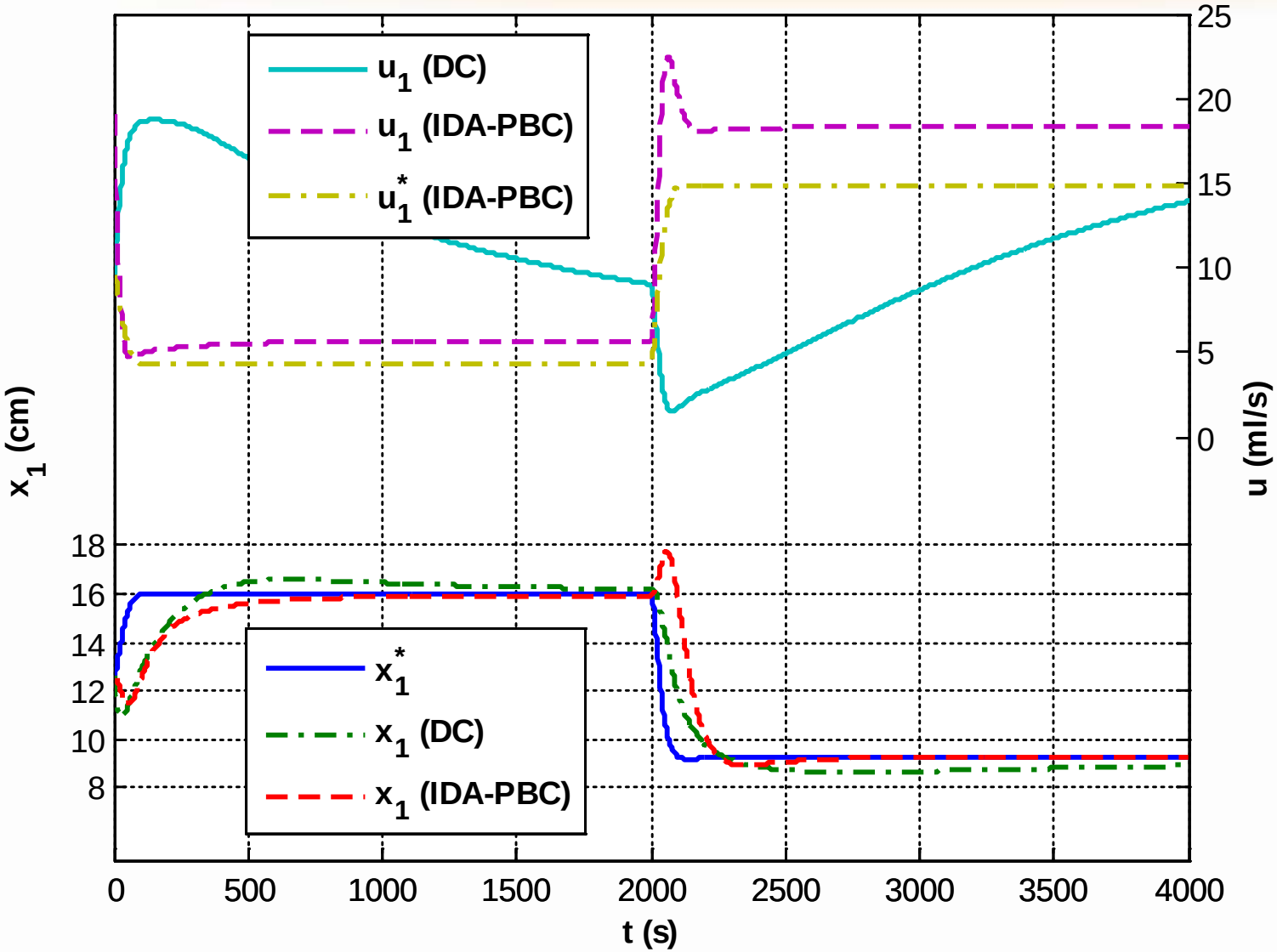
$$\gamma_1 = 0.7$$

$$\gamma_2 = 0.6$$

$$k_u = 0.8$$

$$T_s = 0.1 \text{ s}$$

Four Tank Process Step Response (nonmin-phase)



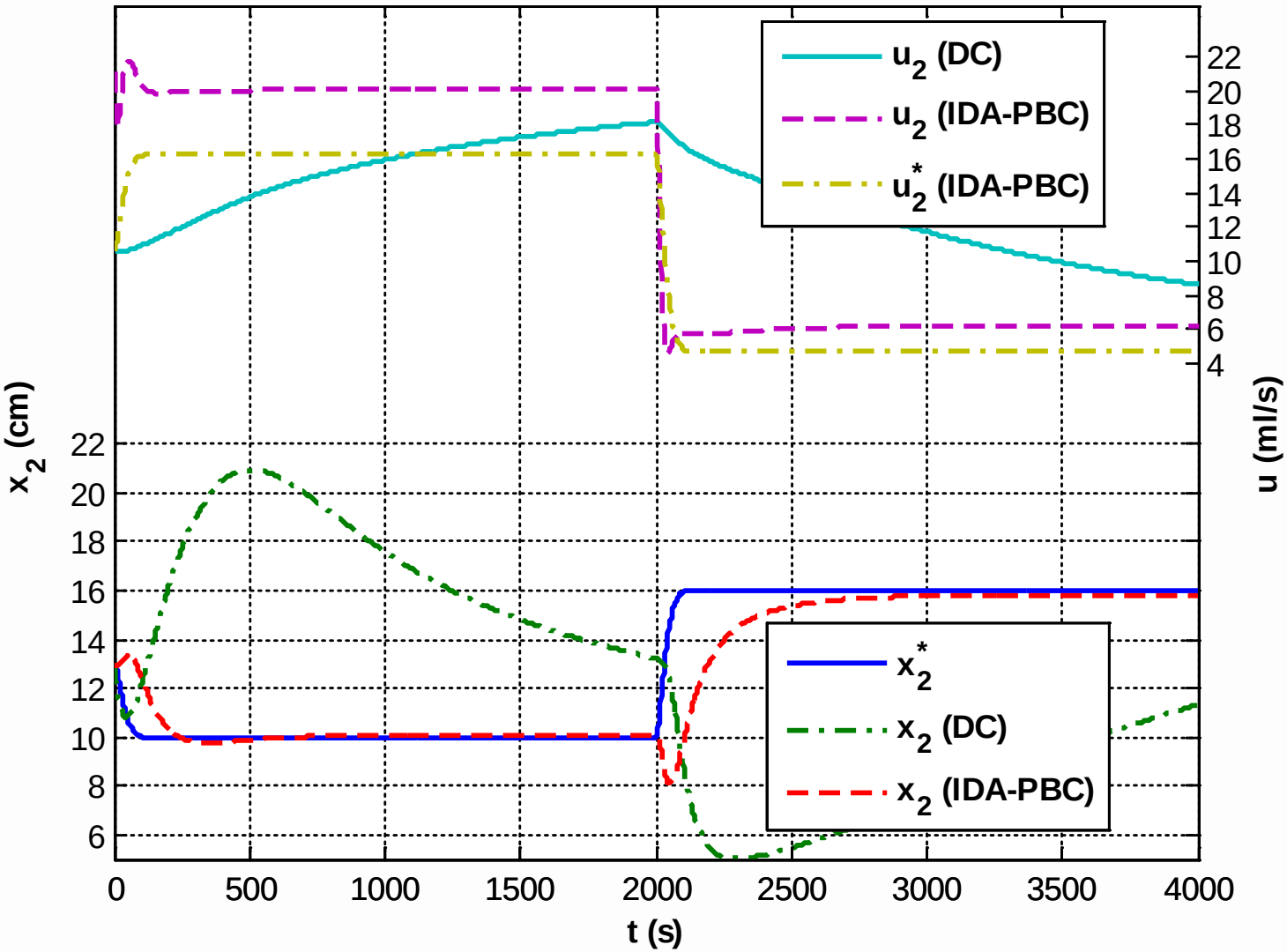
$$\gamma_1 = 0.43$$

$$\gamma_2 = 0.34$$

$$k_u = 0.8$$

$$T_s = 0.1 \text{ s}$$

Four Tank Process Step Response (nonmin-phase)



$$\gamma_1 = 0.43$$

$$\gamma_2 = 0.34$$

$$k_u = 0.8$$

$$T_s = 0.1 \text{ s}$$

Conclusions

- Once correct constraints for the control gains for the control of the nonminimum phase system were determined the IDA-PBC was superior to the decentralized controller option.
- We provided a feasible integrator antiwindup compensator to improve system resilience for the two-tank and four-tank systems studied.
- The explicit solution for control trajectories under ideal model conditions can provide both visual feedback to the operator indicating drifts from these ideal conditions and can be potentially used to determine system faults.
- The four tank process appears to be sufficiently rich to integrate with various digital control infrastructures in order to evaluate various red-team/ blue-team scenarios.
- Further work needs to be taken to address integrator antiwindup compensator for nonlinear systems and to attempt more explicit guarantees for discrete-time control systems which rely on IDA-PBC techniques.