A Modern Introduction to Static and Dynamic Non-cooperative Game Theory

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5th International Symposium on Resilient Control Systems

Aug. 14, 2012

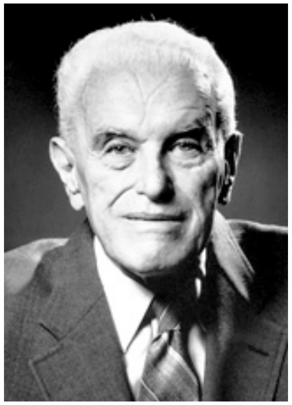
Outline

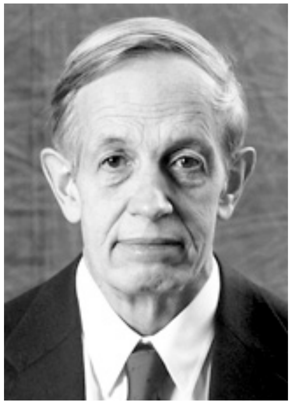
- Introduction
- Static Games
  - Matrix Games
  - Learning in Games
  - Stackelberg Games
  - Games-in-Games
- Dynamic Games
  - Extensive Games
  - Differential Games
  - PoA and PoI
  - Large Population Games
- Summary

Game Theory

- Quantitative methods for strategic interactions between entities/players
- 65+ years of scientific development
- 8 Nobel Prizes (1994/2005/2007)
  - 1994: John Harsanyi, John Nash, Reinhard Selten

    "for their pioneering analysis of equilibria in the theory of non-cooperative games"

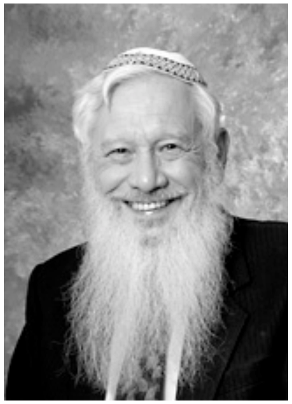




Game Theory

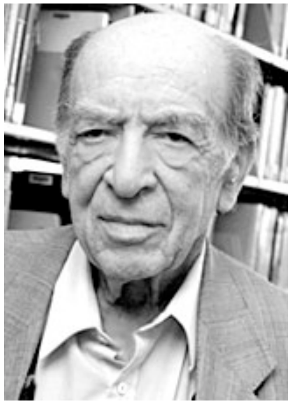
- Quantitative methods for strategic interactions between entities/players
- 65+ years of scientific development
- 8 Nobel Prizes (1994/2005/2007)
  - 2005: Robert Aumann, Thomas Schelling

    “for having enhanced our understanding of conflict and cooperation through game-theory analysis”



Game Theory

- Quantitative methods for strategic interactions between entities/players
- 65+ years of scientific development
- 8 Nobel Prizes (1994/2005/2007)
  - 2007: Leonid Hurwicz, Eric Maskin, Roger Myerson
    “for having laid the foundations of mechanism design theory”



Game Theory

- Quantitative methods for strategic interactions between entities/players
- 65+ years of scientific development
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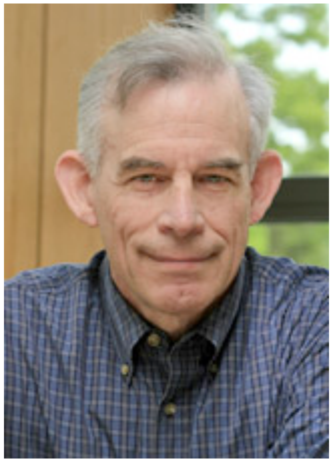
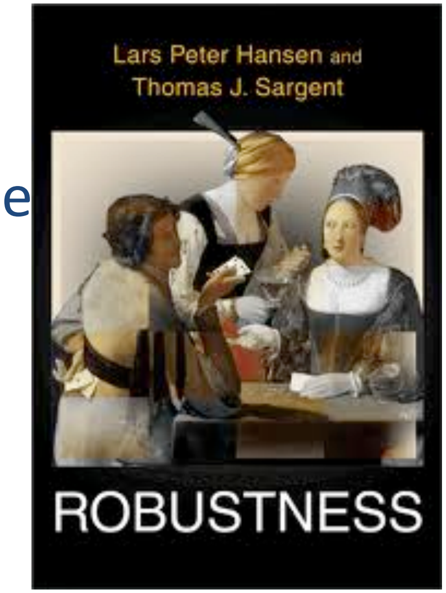
- Recent Nobel Prize 2011
  - Thomas J. Sargent and Christopher A. Sims
    "for their empirical research on cause and effect in the macroeconomy"



Game Theory



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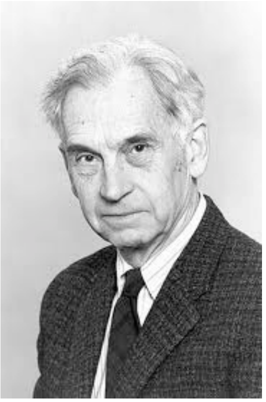
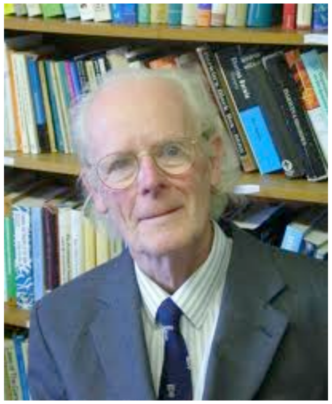
nal expectations to work. When we became aware of Whittle's 1990 book, *Risk Sensitive Control*, and later his 1996 book, *Optimal Control: Basics and Beyond*, we eagerly worked through them. These and other books on robust control theory, such as Başar and Bernhard's 1995 H^∞– *Optimal Control and Related Minimax Design Problems: A Dynamic Game Approach*, provide tools for approaching the 'soft' but important question of how to make decisions when you don't fully trust your model.

Game Theory

- Quantitative methods for strategic interactions between entities/players
- 65+ years of scientific development
- 8 Nobel Prizes (1994/2005/2007)
- Recent Nobel Prize 2011
- Crafoord Prize (1999)
  - Ernst Mayr, John Maynard Smith and George Williams "for developing the concept of evolutionary biology"





Game Theory

- Quantitative methods for strategic interactions between entities/players
- 65+ years of scientific development
- 8 Nobel Prizes (1994/2005/2007)
- Recent Nobel Prize 2011
- Crafoord Prize (1999)

- Applications in various fields
  - Auction theory
  - Network science
  - Control theory
  - Security science
  - Transportation
  - Biology, etc.

Game Theory: Present

- Societies
  - International Society of Dynamic Games (1990 -)
  - Game Theory Society (1999 -)
  - Several regional ones

- Conferences and Symposia (numerous)

- Book Series
  - T. Başar (Series Ed.), *Static & Dynamic Game Theory: Foundations & Applications*, Birkhäuser, 2011

- Journals (Numerous)
  - Games and Economic Behavior
  - International J. Game Theory
  - J. Dynamic Games and Applications

Game Theory: Present

- Books
  - G. Owen, *Game Theory*, 3rd edition, AP, 1995
  - D. Fudenberg, J. Tirole, *Game Theory*, MIT Press, 1991
  - T. Başar, G.J. Olsder, *Dynamic Noncooperative Game Theory*, 2nd edition, SIAM Classics, 1999
  - R.B. Myerson, *Game Theory: Analysis of Conflict*, Harvard, 1991
  - M. J. Osborne, A. Rubinstein, *A Course in Game Theory*, MIT Press, 1994.

- Books (lighter side and more specialized)
  - K. Binmore, *Fun and Games*, D.C. Heath and Co, 1992
  - J.D. Williams, *The Compleat Strategyst*. McGraw-Hill, 1954
  - W. Poundstone, *Prisoner's Dilemma*, Doubleday, 1992

Game Theory: Rich in Models and Concepts

- Zero-sum vs. Nonzero-sum games
- Non-cooperative vs. Cooperative games
- Complete vs. Incomplete information games
- Deterministic vs. Stochastic games
- Static vs. Dynamic/Differential games
- Stackelberg games
- Multi-layer and multi-resolution games
- Large population games
- Bargaining, bidding, auctions, …

Static Games

- Matrix Games
- Learning in Games
- Stackelberg Games
- Games-in-Games

Generic Non-Cooperative Games

- Players: $\mathcal{N} = \{1, 2, \ldots, N\}$
  - Decision/action for Player i: $x_i \in X_i$.
  - Possible coupled constraints: $\mathbf{x} \in \Omega \subseteq \mathbf{X}$.
  - Net utility function for each player: $V_i(x_i, x_{-i})$.

  - x_{-i} : decisions/actions of all players other than Player i
  - V_i is maximized by Player i over $\Omega(x_{-i})$
  - Game triplet: $\langle \mathcal{N}, \{V_i\}_{i \in \mathcal{N}}, \{\Omega(x_{-i})\}_{i \in \mathcal{N}} \rangle$

Equilibrium of Generic Non-Cooperative Games

- Players: $\mathcal{N} = \{1, 2, \ldots, N\}$
  - Decision/action for Player i: $x_i \in X_i$.
  - Possible coupled constraints: $\mathbf{x} \in \Omega \subseteq \mathbf{X}$.
  - Net utility function for each player: $V_i(x_i, x_{-i})$.

- Non-cooperative Nash Equilibrium (NE): $\mathbf{x}^*$

$$V_i(x_i^*, x_{-i}^*) \geq V_i(x_i, x_{-i}^*), \text{ for all } x_i \in X_i, x_i \in \Omega(x_{-i}), i \in \mathcal{N}.$$

- Players can not benefit by unilaterally deviating from their strategies.

Equilibrium of Generic Non-Cooperative Games

- Players: $\mathcal{N} = \{1, 2, \ldots, N\}$
  - Decision/action for Player i: $x_i \in X_i$.
  - Possible coupled constraints: $\mathbf{x} \in \Omega \subseteq \mathbf{X}$.
  - Net utility function for each player: $V_i(x_i, x_{-i})$.

- Zero-sum game: $N = 2$, $V := -V_1 = V_2$
  - NE is Saddle-Point (SP).

$$V(x_1^*, x_2) \leq V(x_1^*, x_2^*) \leq V(x_1, x_2^*)$$

Example 1: Prisoner's Dilemma

(G_1)

	C	D
C	2 , 2	0 , $\underline{3}$
D	$\underline{3}$, 0	$\underline{1}$, $\underline{1}$

- Both players are maximizers.
- NE in pure strategies (D, D) vs. socially optimal solution (C, C)
- Loss of efficiency:

$$\text{Price of Anarchy (PoA)} = \frac{\text{Social Welfare under NE}}{\text{Optimal Social Welfare}} = \frac{1+1}{2+2} = 50\%$$

- Decentralization: Resilience vs. Robustness

Example 2: Battle of Sexes

(G_2)		B	S
	B	2 , 1	0 , 0
	S	0 , 0	1 , 2

- Both players are maximizers.
- B = Bach (or Ballet); S = Stravinsky (or Soccer)
- Two NE in pure strategies: (B, B) and (S, S)
- One NE in mixed strategy: {(2/3, 1/3), (1/3,2/3)}
- It is a strategic game of cooperation (Interests are aligned).
- Win-win vs. win-lose situations.

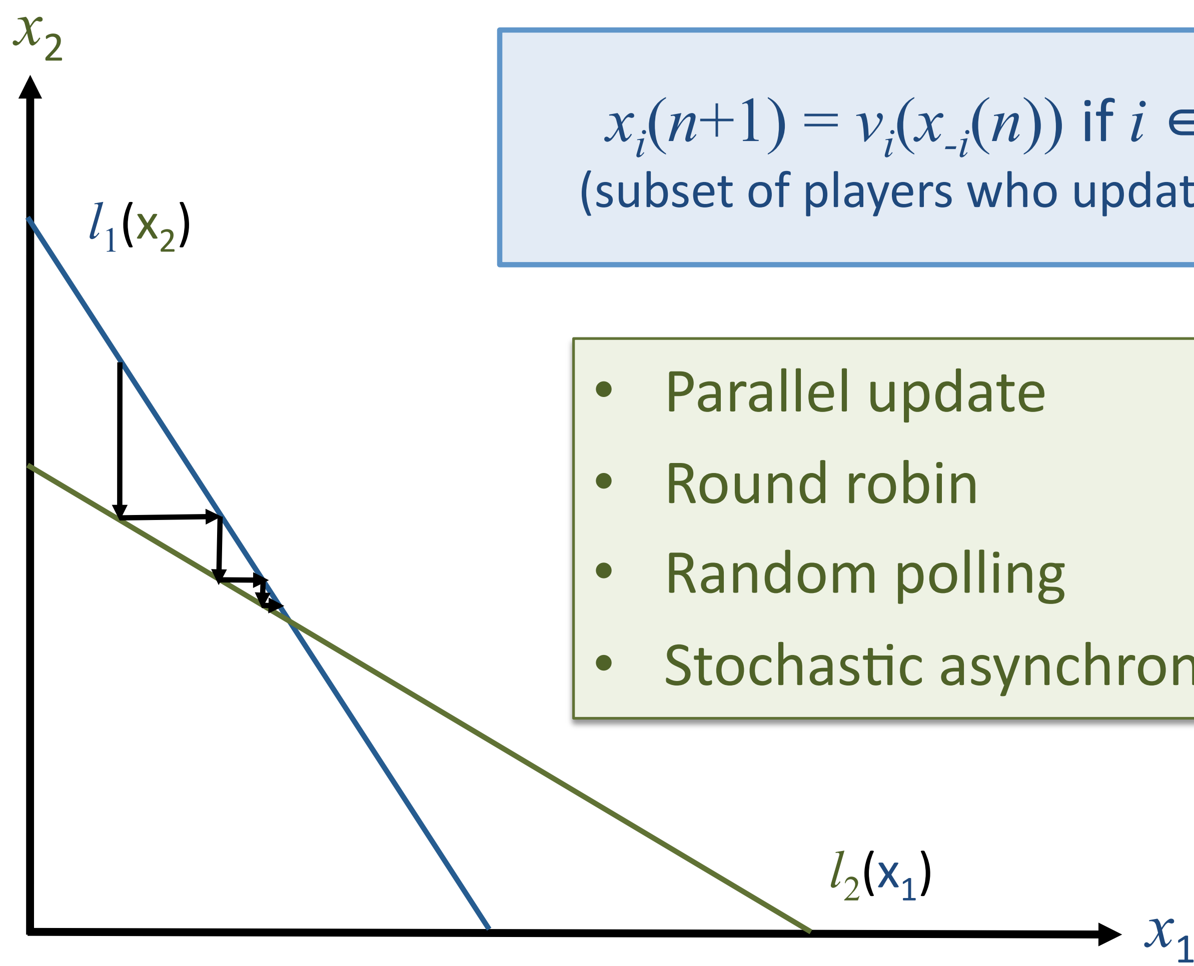
Example 3: Matching Penny Game

(G_3)

	H	T
H	1 , -1	-1 , 1
T	-1 , 1	1 , -1

- Both players are maximizers.
- No existence of pure strategy equilibrium
- SP equilibrium in mixed strategies: {(0.5, 0.5), (0.5, 0.5)}
- Value of the game: $val[G_3] = 0$
- Every finite zero-sum matrix game has a SPE in mixed strategies – Minimax Theorem (Von Neumann 1928)
- Attacker vs. Defender
- Disturbances vs. Robust control

Iterative Algorithms: Best Response Algorithm



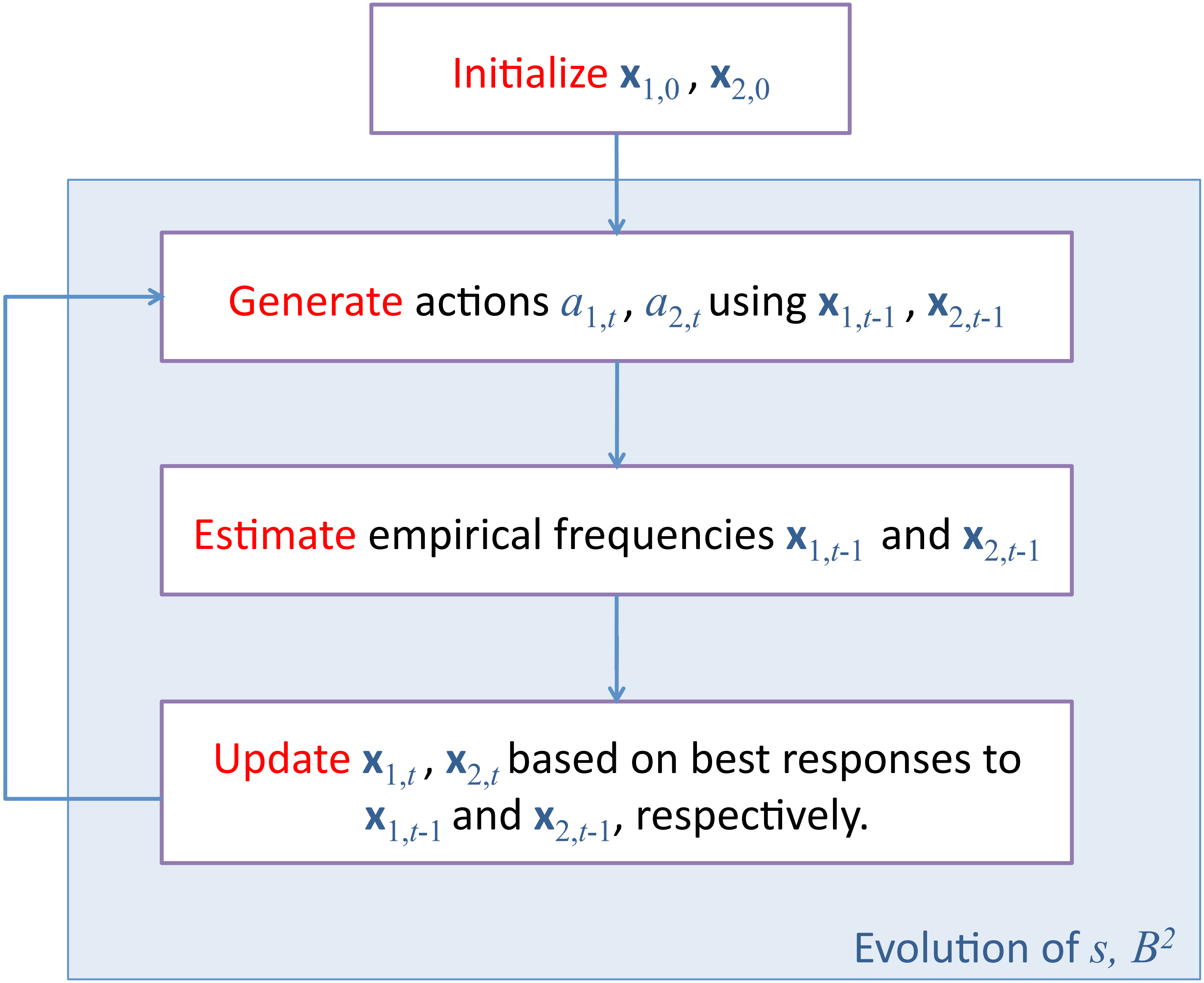
$$x_i(n+1) = v_i(x_{-i}(n)) \text{ if } i \in K_n$$

(subset of players who update at n)

- Parallel update
- Round robin
- Random polling
- Stochastic asynchronous

Learning Algorithms

- Learning algorithms are essential for applications of game theory.
- Classical learning algorithms
  - Best response dynamics
  - Fictitious play
- A new class of learning algorithms
  - No knowledge of your own payoff function
  - No knowledge of the payoff function of your opponents
  - No knowledge of the action spaces of the opponents
- Players have different levels of rationality and intelligence
  - Active learner vs. passive learner
  - Fast learner vs. slow learner
  - Homogeneous learn vs. heterogeneous and hybrid learner
- Players do not interact all the time.

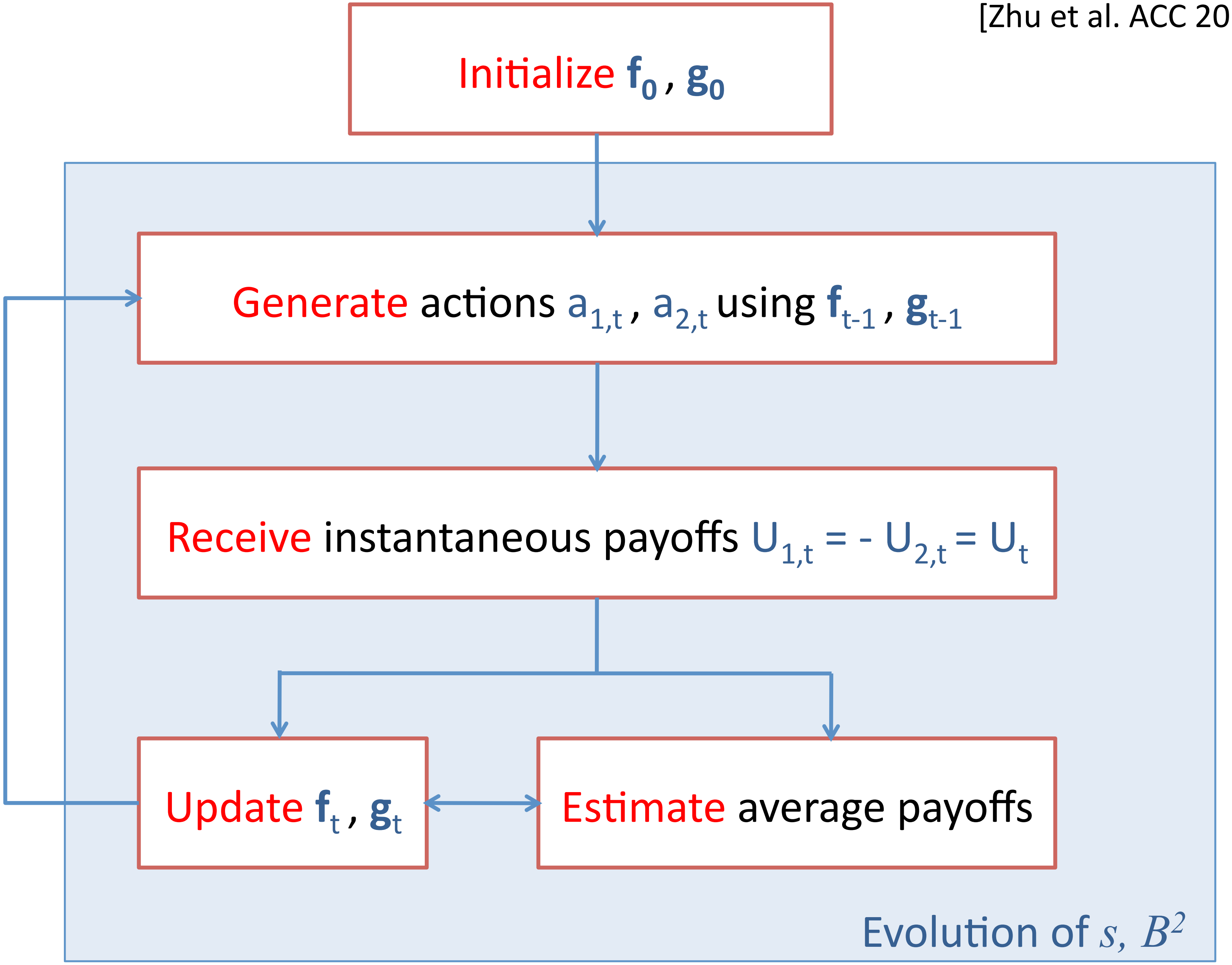

Initialize $\mathbf{x}_{1,0}$, $\mathbf{x}_{2,0}$
Generate actions $a_{1,t}$, $a_{2,t}$ using $\mathbf{x}_{1,t\text{-}1}$, $\mathbf{x}_{2,t\text{-}1}$
Estimate empirical frequencies $\mathbf{x}_{1,t\text{-}1}$ and $\mathbf{x}_{2,t\text{-}1}$
Update $\mathbf{x}_{1,t}$, $\mathbf{x}_{2,t}$ based on best responses to $\mathbf{x}_{1,t\text{-}1}$ and $\mathbf{x}_{2,t\text{-}1}$, respectively.
Evolution of s, B^2

Fictitious-Play Algorithms

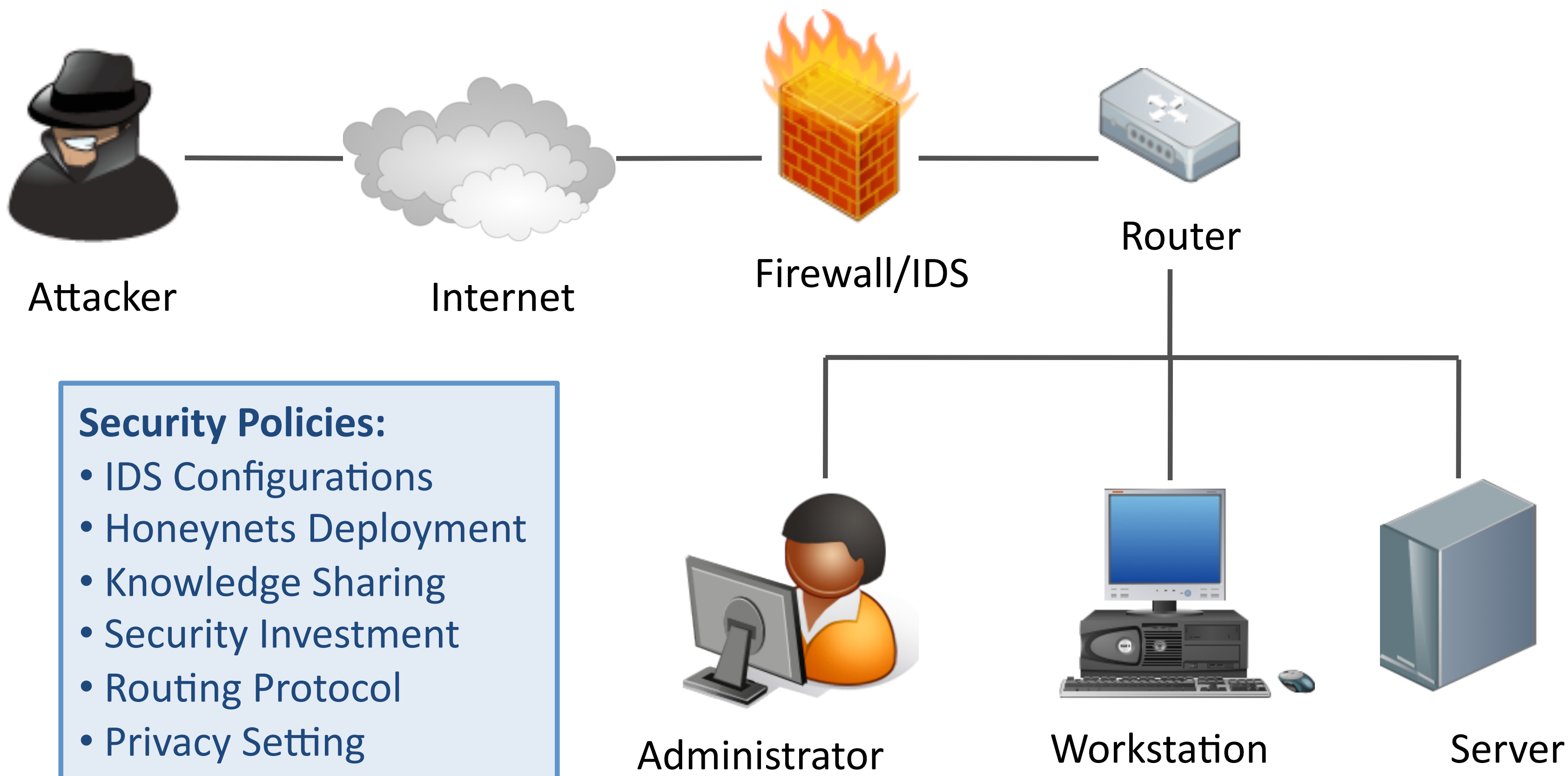
Row Player \ Column Player	H	T
H	1 , -1	-1 , 1
T	-1 , 1	1 , -1

Round t	Row Action	Column Action	Row Empirical Counts	Column Empirical Counts	Row Strategy	Column Strategy
0			(4/5,1/5)	(1/5,4/5)	(1, 0)	(1, 0)
1	H	H	(5/6, 1/6)	(2/6, 4/6)	(0,1)	(0, 1)
2	T	T	(5/7, 2/7)	(2/7, 5/7)	(0,1)	(0,1)

[Zhu et al. CDC 2010]
[Zhu et al. ACC 2011]



Applications to Network Security



Security Policies:
- IDS Configurations
- Honeynets Deployment
- Knowledge Sharing
- Security Investment
- Routing Protocol
- Privacy Setting

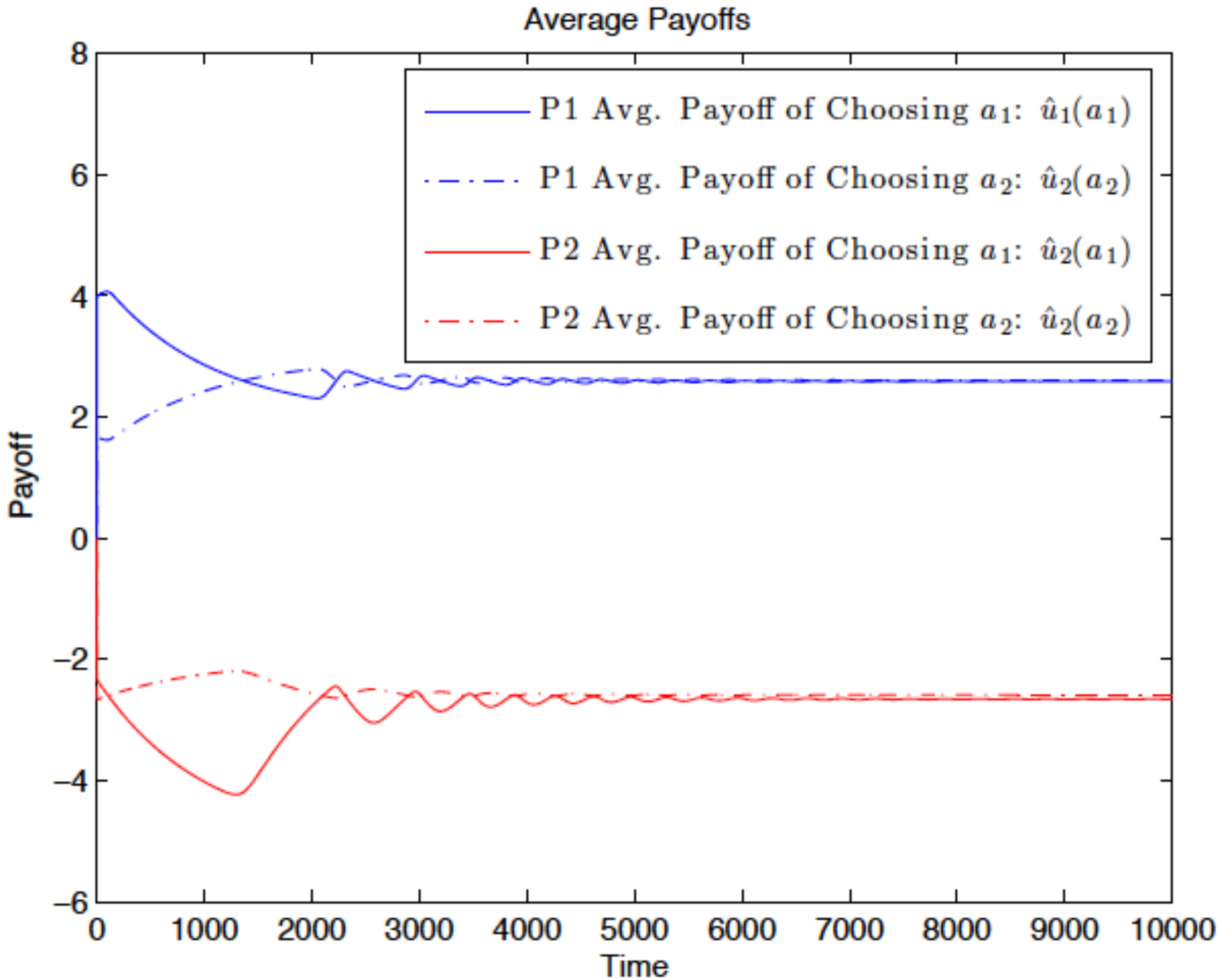
Network Security: A Simple Model

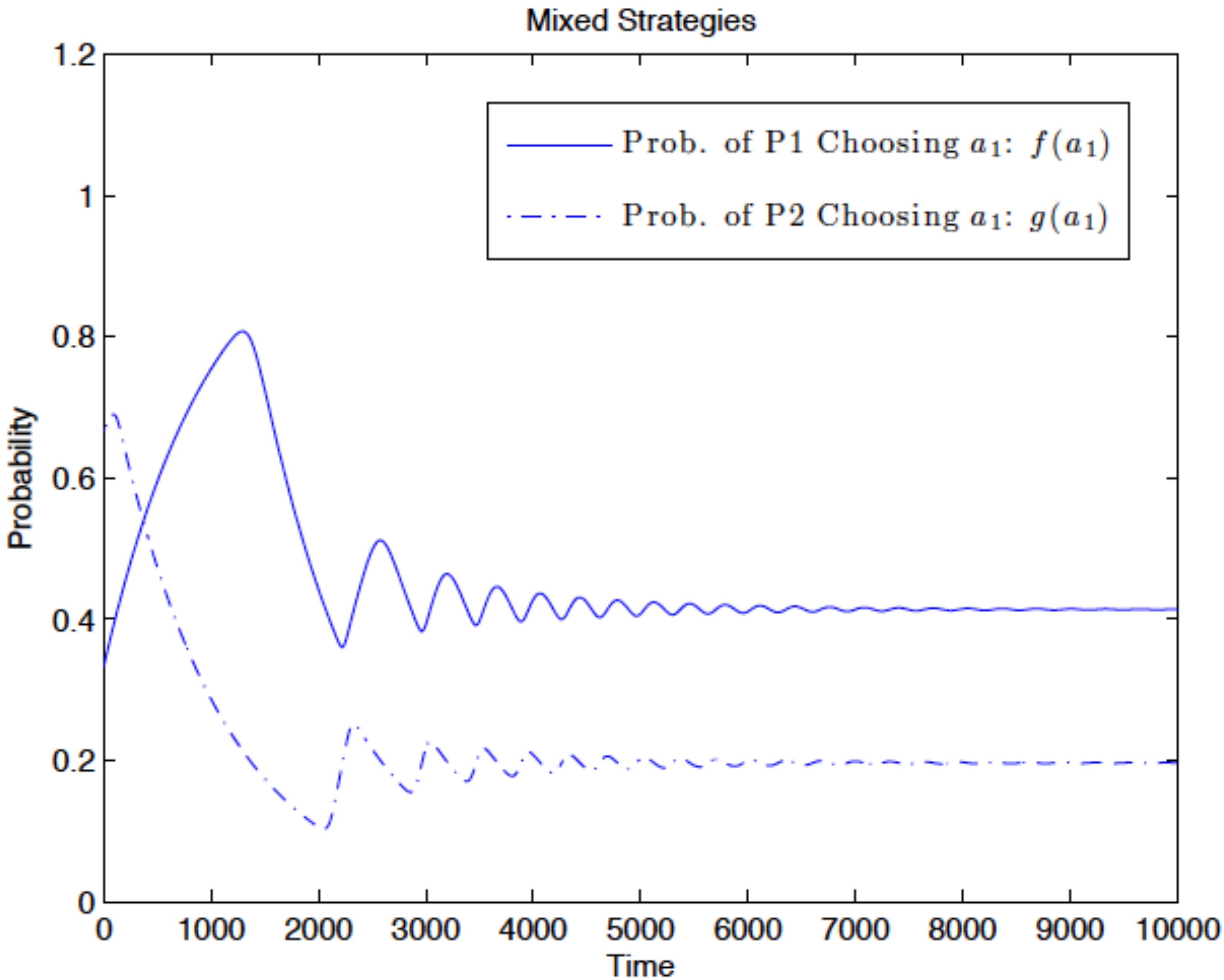
- Consider a two-person game:
  - The defender (P1) and the attacker (P2)
  - P1 (row player): either to defend (D) or not to defend (ND).
  - P2 (column player): either to attack or not to attack (NA).

	N	NA
D	5 , -5	2 , -2
ND	1, -1	3 , -3

+ Noise

- Knowledge of P1 and P2:
  - Players do not know the payoff matrix.
  - Players do not have the knowledge of the action spaces of each others.


Average Payoffs
Payoff
Time
P1 Avg. Payoff of Choosing a_1: $\hat{u}_1(a_1)$
P1 Avg. Payoff of Choosing a_2: $\hat{u}_2(a_2)$
P2 Avg. Payoff of Choosing a_1: $\hat{u}_2(a_1)$
P2 Avg. Payoff of Choosing a_2: $\hat{u}_2(a_2)$
8
6
4
2
0
−2
−4
−6
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000

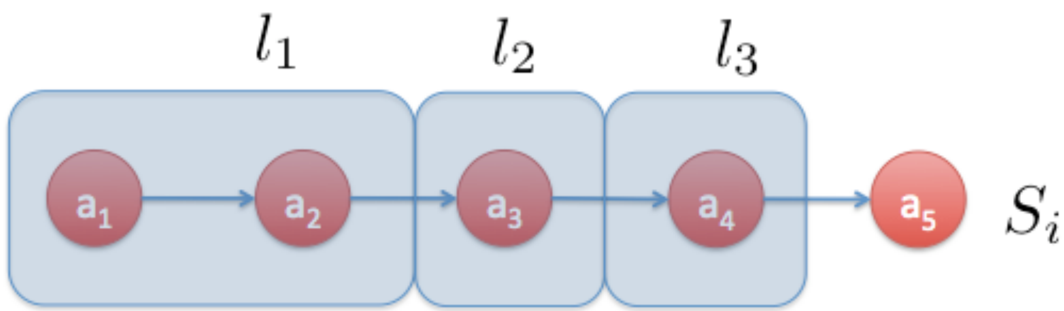


Mixed Strategies
Prob. of P1 Choosing a_1: $f(a_1)$
Prob. of P2 Choosing a_1: $g(a_1)$
Probability
Time
1.2
1
0.8
0.6
0.4
0.2
0
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000

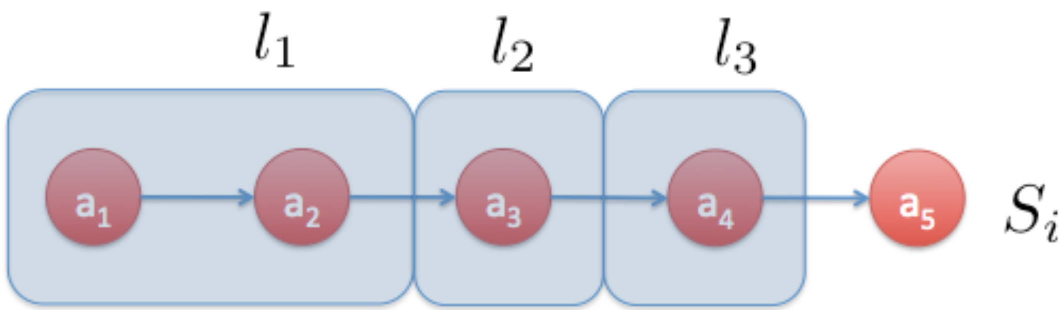
Network Security: IDS Configuration

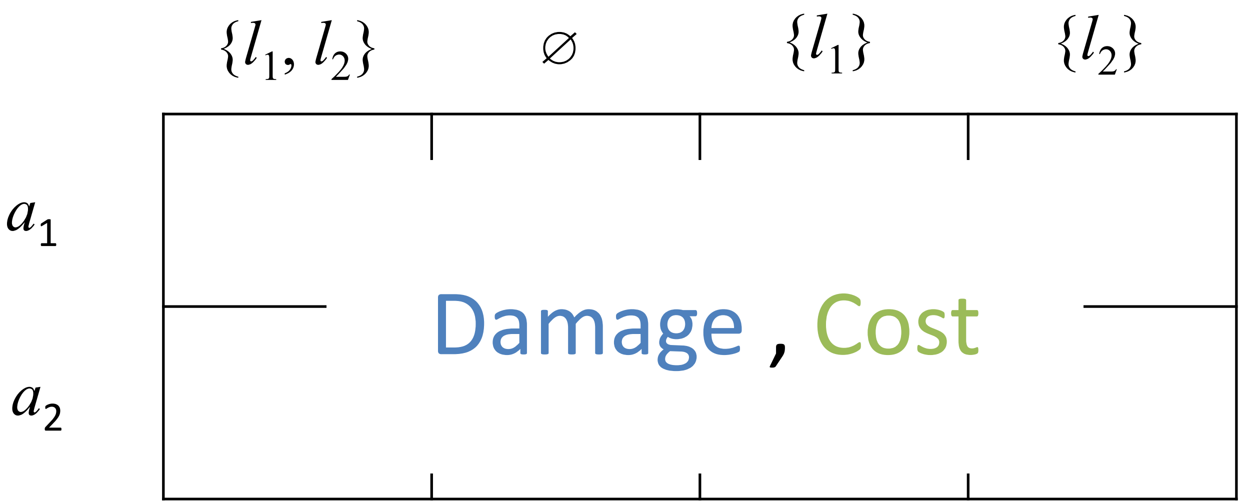
System Model: An Example

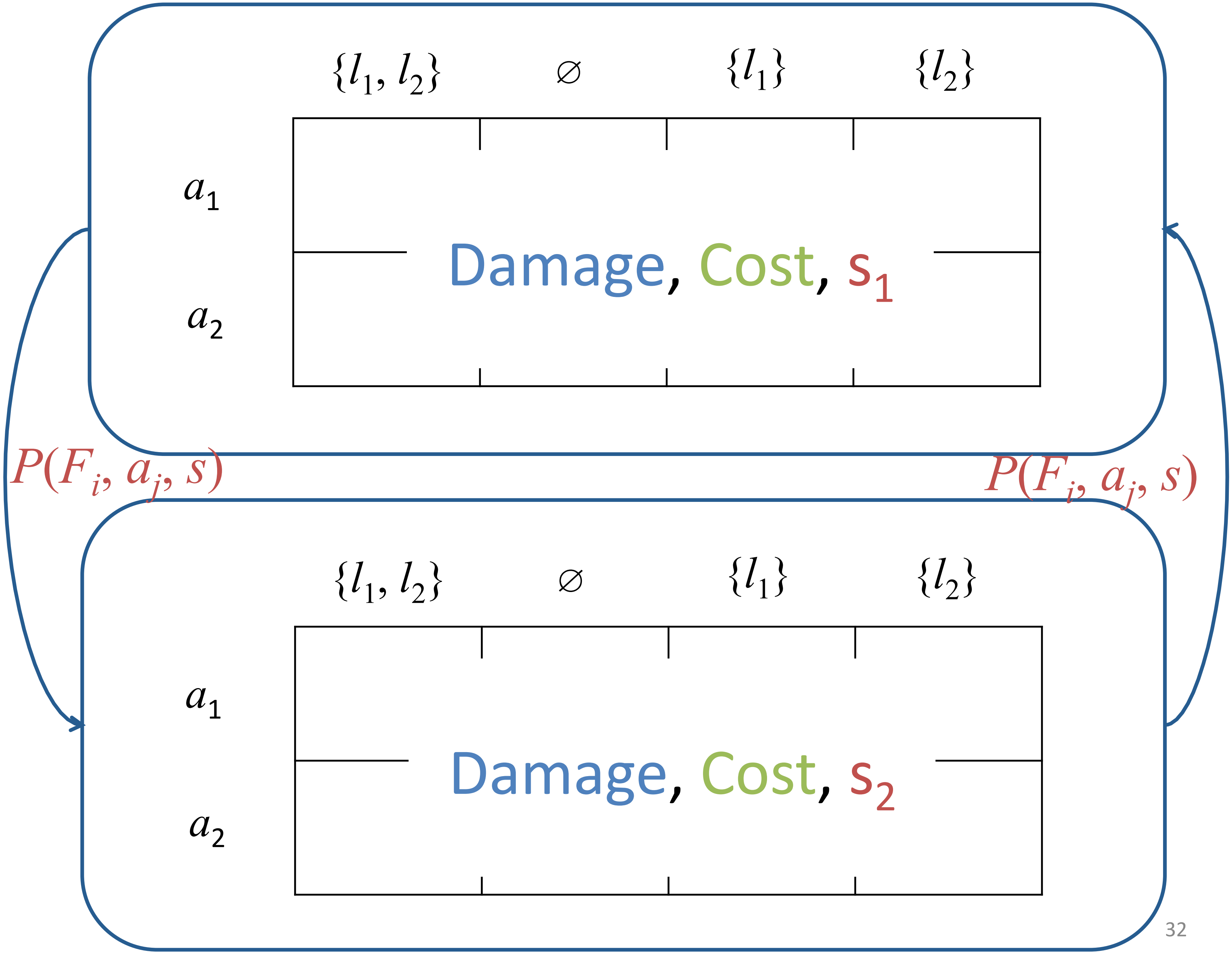
- IDS Model
  - 2 libraries $\mathcal{L} = \{l_1, l_2\}$
  - 4 configurations (can be constrained): $\{\{l_1, l_2\}, \varnothing, \{l_1\}, \{l_2\}\}$
  - An IDS chooses an optimal configuration F_i.
- Attacker Model
  - An attacker has different types of attack a_1, a_2, ..., a_M.
  - An attacker chooses a sequence of attacks, e.g. from attack trees, $S_i = \{a_1, a_2, a_3, a_4, a_5\}$.



System Model: An Example






$\{l_1, l_2\}$
$\varnothing$
$\{l_1\}$
$\{l_2\}$
a_1
a_2
Damage, Cost, s_1
$P(F_i, a_j, s)$
$P(F_i, a_j, s)$
$\{l_1, l_2\}$
$\varnothing$
$\{l_1\}$
$\{l_2\}$
a_1
a_2
Damage, Cost, s_2

Stackelberg Game

- NE is efficient if it is also Pareto optimal solution or it maximizes $\Sigma_i V_i (x_i, x_{-i})$.

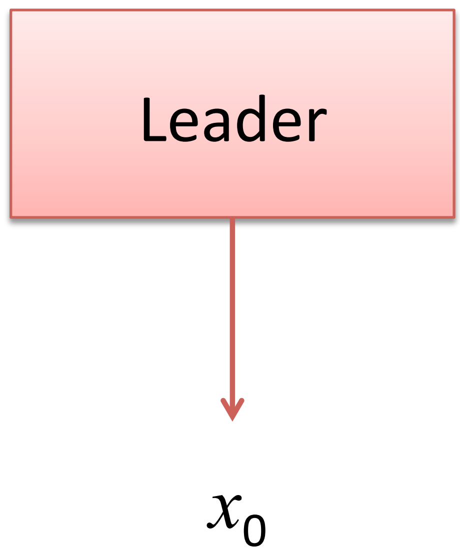
- One way to induce efficiency is through incentives (e.g. pricing): Obtain efficient NE of the game with individual payoffs by choosing proper $\{r_i\}$.

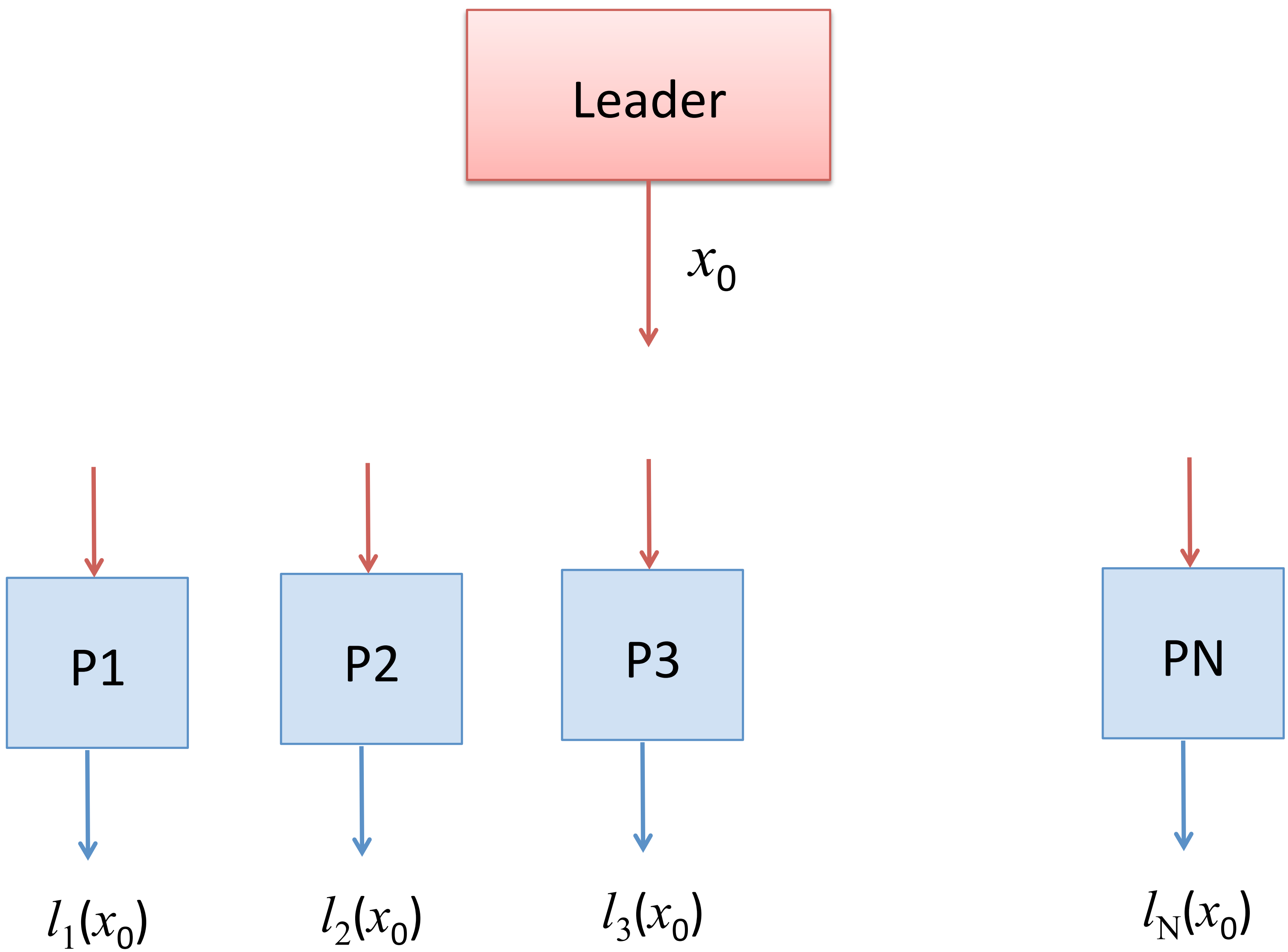
$$V_i (x_i, x_{-i}) = U_i (\mathbf{x}) - r_i(x_i)$$

- Essentially a Stackelberg game where $\{r_i\}$ are leaders' decision variables.

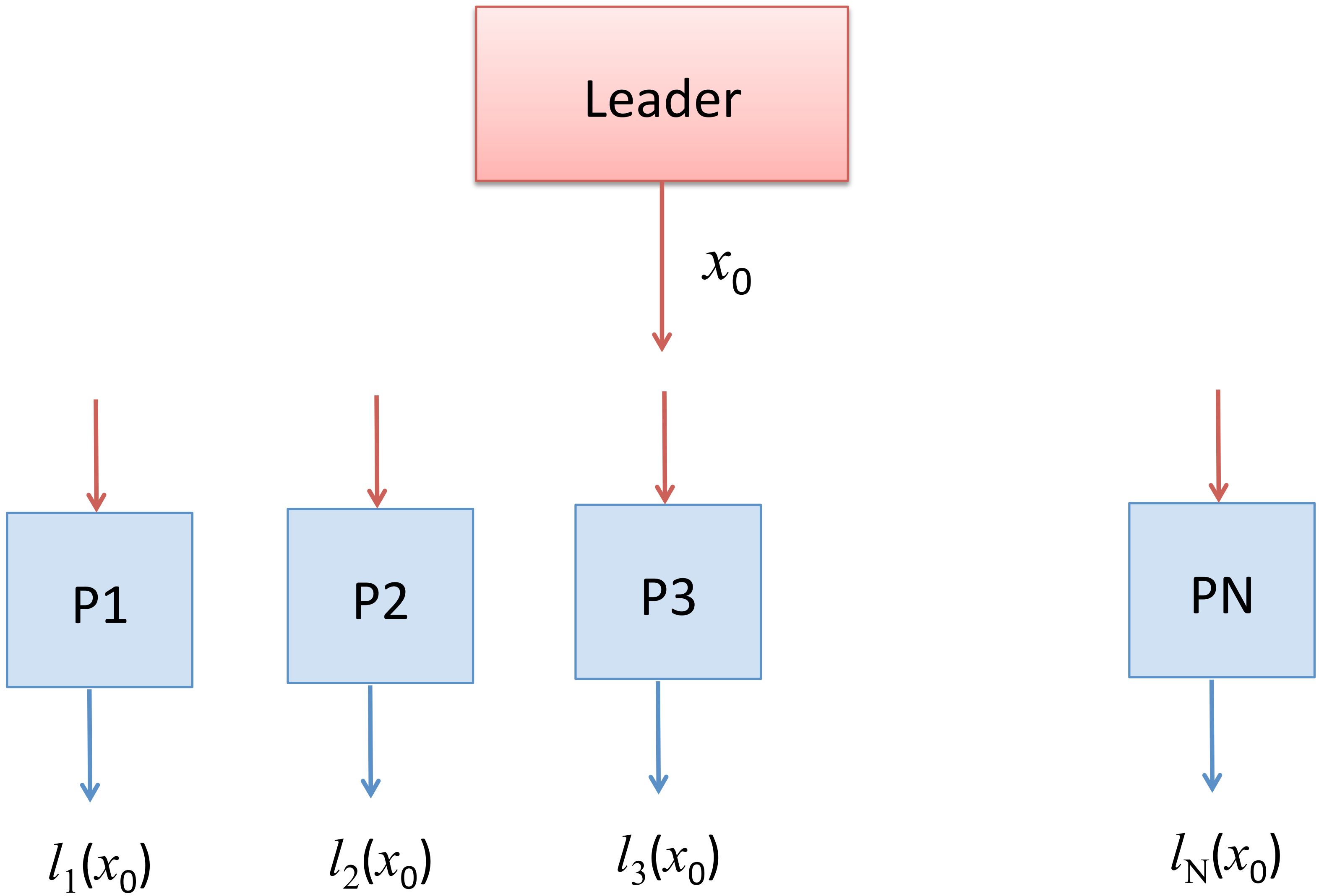
Stackelberg Solution Concept

Leader announces an action/strategy




Leader
x_0
P1
P2
P3
PN
$l_1(x_0)$
$l_2(x_0)$
$l_3(x_0)$
$l_N(x_0)$

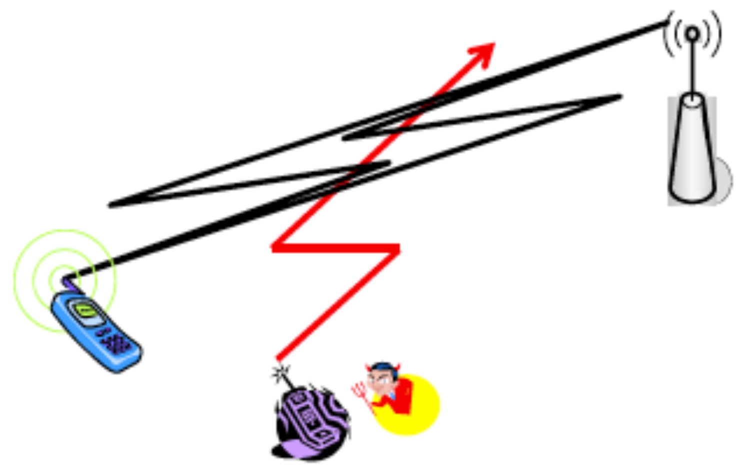
$$V_0(x_0^*; l(x_0^*)) = \max V_0(x_0; l(x_0))$$



Security Games

Defender \ Attacker	Active	Passive
Active	Nash	Stackelberg
Passive	Stackelberg	Nash

Eavesdropper



Jammer

[Manshaei et al., ACM Survey, 2012]

- In addition to N players, Player 0 as leader, with utility $V_0(x_0; x_1,.., x_N)$, who chooses an action/strategy x_0 to maximize his utility.

- $\boldsymbol{x}^*$ a Stackelberg equilibrium solution (SES) if

$$\boxed{V_0(x_0^*; l(x_0^*)) = \max V_0(x_0; l(x_0))}$$

  - $l(x_0)$ is unique NE solution of the N-person follower game.

- No general clean existence and uniqueness results for multi-follower Stackelberg solution.

- For two-person NZS games, if X_1 and X_2 are compact, $V_i(x_1, x_2)$ is continuous, $i = 1, 2$, and $l_2(x_1)$ is characterized by a finite family of continuous maps, then the NZSG admits a SES.

Example

(G_S)

P2

P1		L	M	R
	L	0, -1	2 , 1	3/2 , -2/3
	M	1 , 2	1 , 0	3, 1
	R	-1 , 0	2 , 1	2 , -1/2

- Both players minimize.

Example (Cont'd)

(G_S)

P2

P1		L	M	R
	L	$0, \underline{-1}$	$2, 1$	$\underline{3/2}, -2/3$
P1	M	$1, 2$	$\underline{1}^{N}, \underline{0}^{N}$	$3, 1$
	R	$\underline{-1}, 0$	$2, 1$	$2, \underline{-1/2}$

- Both players minimize.
- (M,M) is the pure-strategy NE with equilibrium cost(1, 0).

Example (Cont'd)

(G_S)

P1 \ P2	L	M	R
L	$0^{S1}, \underline{-1}^{S1}$	$2, 1$	$3/2, -2/3$
M	$1, 2$	$1, \underline{0}$	$3, 1$
R	$-1, 0$	$2, 1$	$2, \underline{-1/2}$

- Both players minimize.
- Player 1 is the leader.
- (L,L) is the Stackelberg equilibrium with cost (0,-1)

Example (Cont'd)

(G_S)

P2

P1		L	M	R
	L	0, -1	2 , 1	$\underline{3/2}^{S2}$, $-2/3^{S2}$
	M	1 , 2	$\underline{1}$, 0	3, 1
	R	$\underline{-1}$, 0	2 , 1	2 , -1/2

- Both players minimize.
- Player 2 is the leader.
- (R,R) is the Stackelberg equilibrium with cost (3/2,-2/3).

Example (Cont'd)

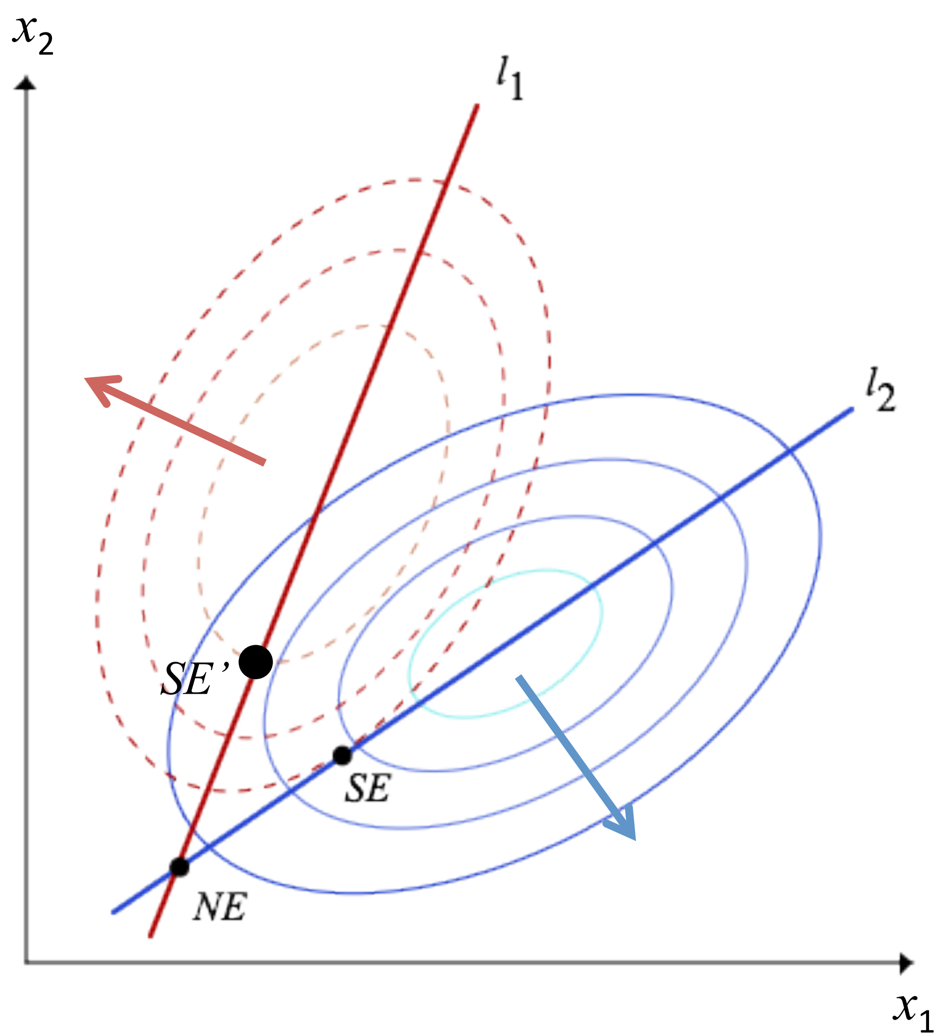
(G_S) P2

P1

	L	M	R
L	$0^{S1}, -1^{S1}$	$2, 1$	$3/2^{S2}, -2/3^{S2}$
M	$1, 2$	$1^{N}, 0^{N}$	$3, 1$
R	$-1, 0$	$2, 1$	$2, -1/2$

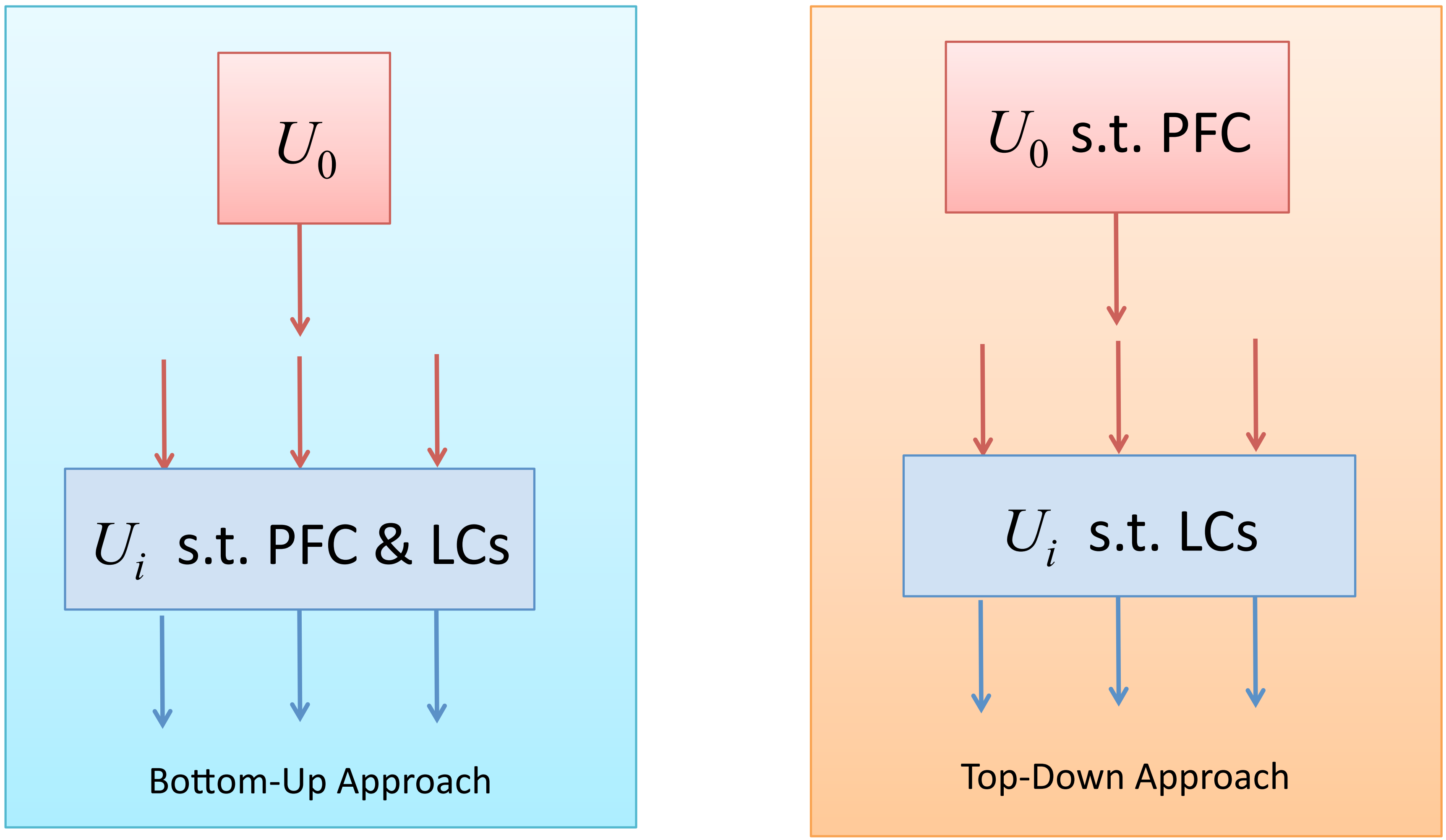
- Both player minimize.
- $(0^{S1}, -1^{S1})$ is better than $(1^{N}, 0^{N})$ for P1 (also for P2).
- $(3/2^{S2}, -2/3^{S2})$ is better than $(1^{N}, 0^{N})$ for P2 (worse for P1).

Graphical Illustrations



- Both players *minimize* their costs J_1 and J_2 (in quadratic forms).
- SE is the Stackelberg equilibrium with P1 as the leader.
- SE yields *lower* costs for both players because the interests of P1 and P2 are *aligned* in some way.
- SE' is the Stackelberg equilibrium with P2 as the leader.
- With unique reaction functions, the leader cannot do worse in SE than in NE.

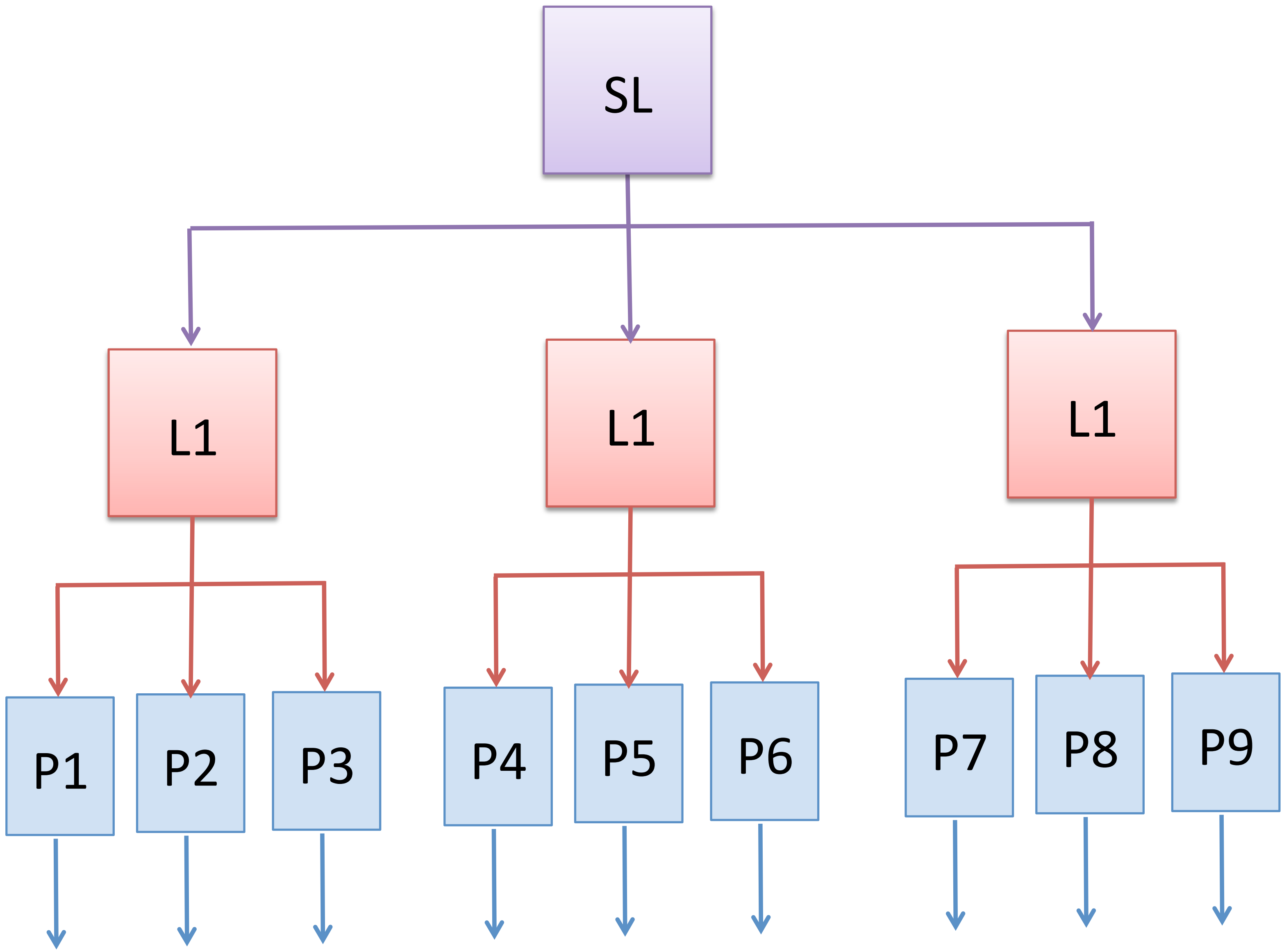
Handling the Constraints: Top-Down vs. Bottom-Up Approaches



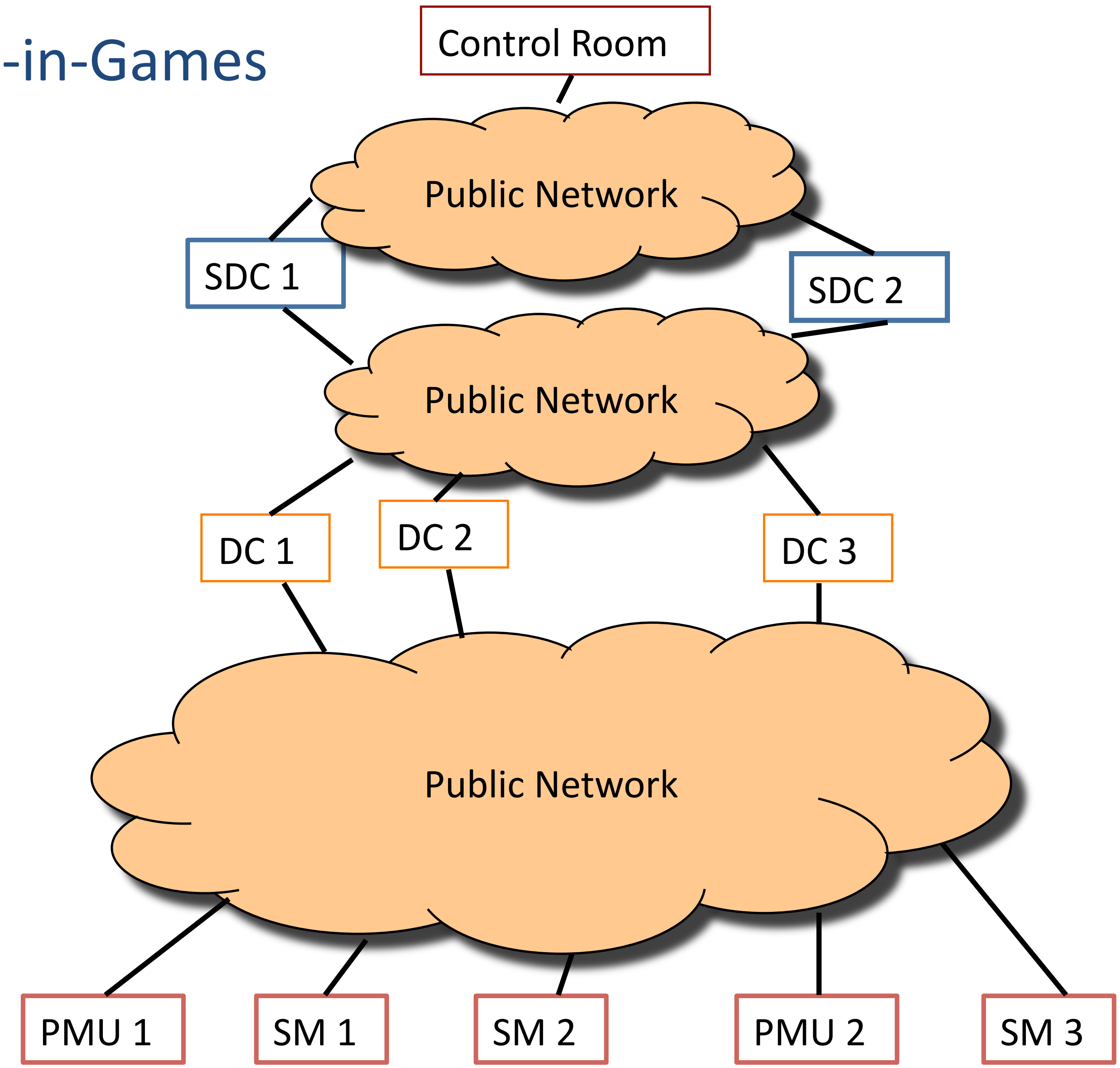
PFC: Power Flow Constraints
LC: Local Constraints

[Zhu and Pavel, 2008]

Multiple Hierarchies

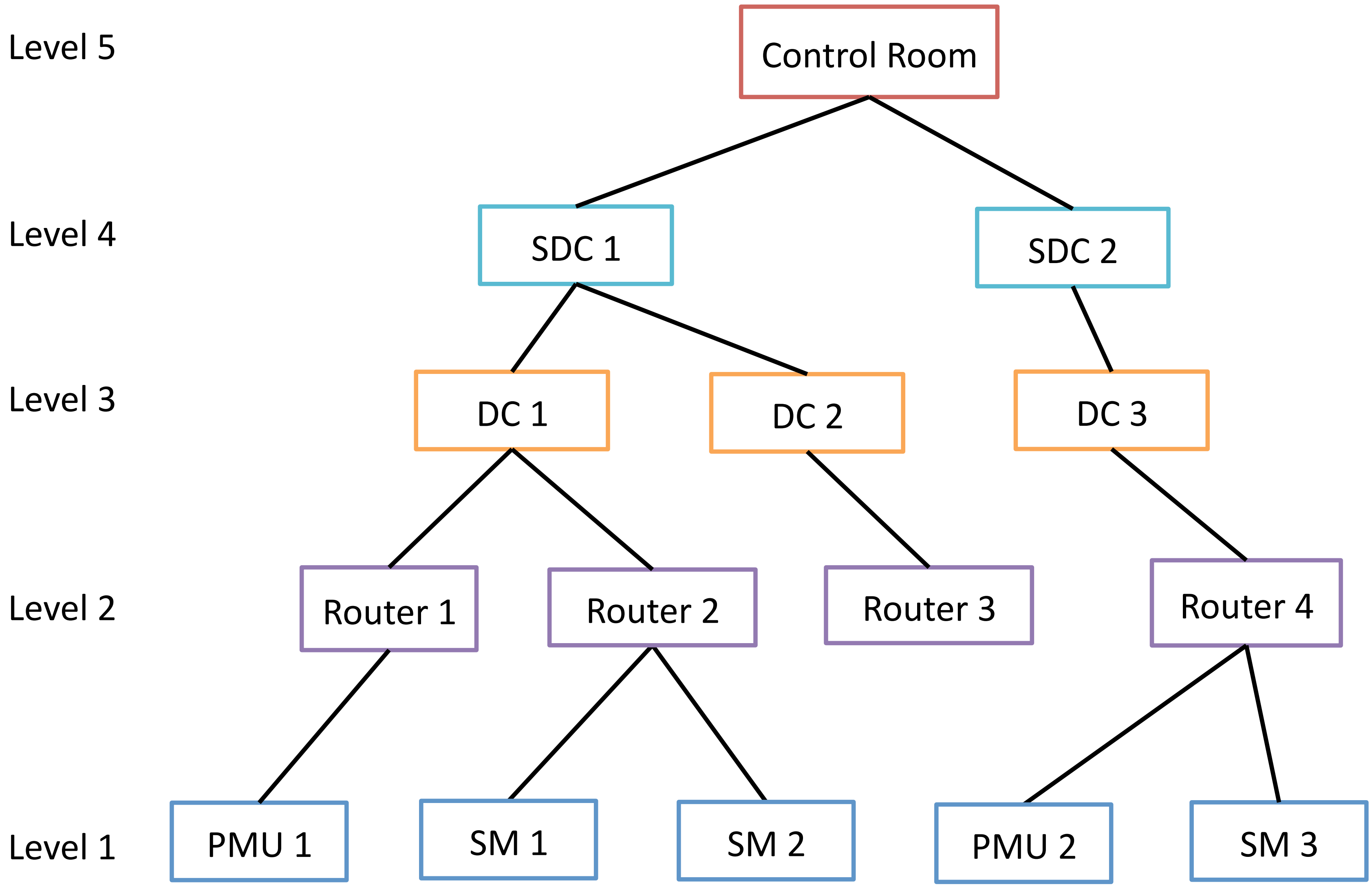


Games-in-Games



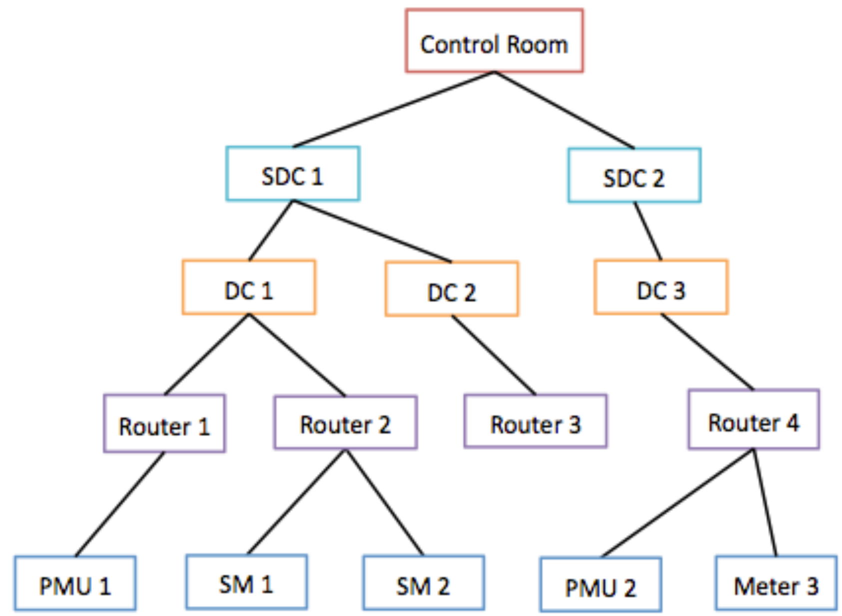
Example: Secure Routing in Smart Grids

Secure Routing in Smart Grids



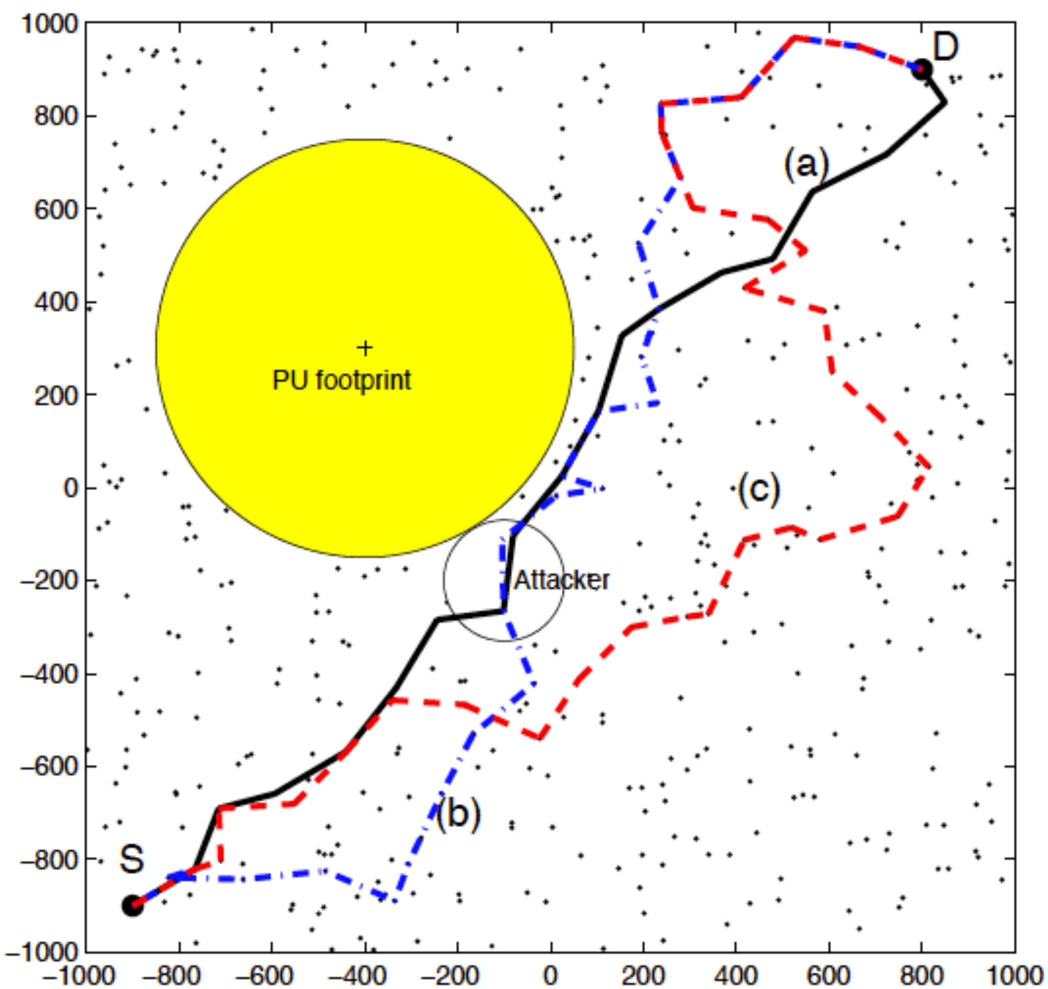
[Zhu et al., CPSWeek 2011 Workshop]

Games-in-Games



[Zhu and Başar, 2011]

Application of Distributed Learning to Games-in-Games Framework in Cognitive Radio Systems



- A secondary user changes its route from the blue line (b) to red line (c) between S and D by learning the presence of a jamming attacker.
- The communication environment changes as the footprint of the primary users

[Zhu et al. Globecom 2011]

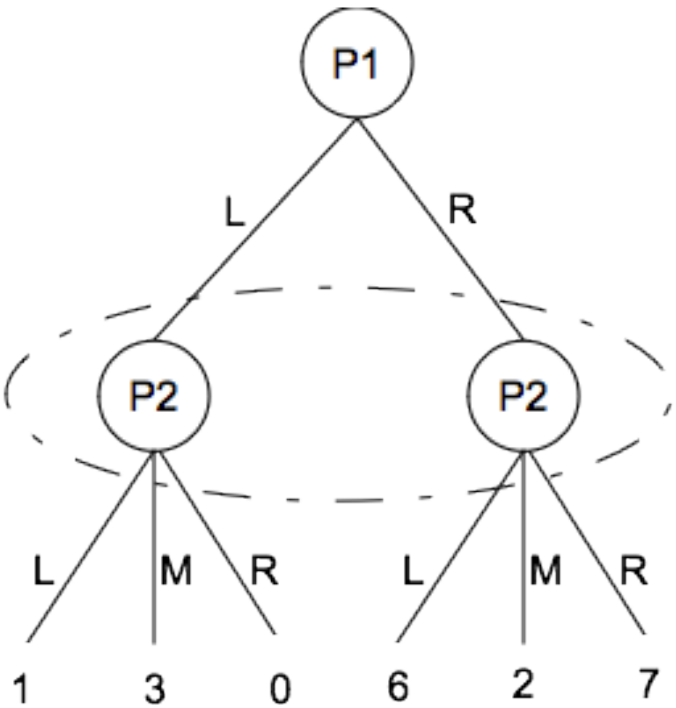
Dynamic Games

- Extensive Games
- Differential Games
- PoA and PoI
- Large Population Games

Dynamic Games

- Repeated games: bargaining, trust games, cooperation, etc.
- Extensive games: chess, poker, etc.
- Differential/Difference games: pursuit-and-evasion games, robust control, etc.
- Evolutionary games: mutations, learning theory, etc.
- Stochastic games
  - Competitive MDPs
  - Stochastic differential/difference games
  - Mean-field games
  - Hybrid games

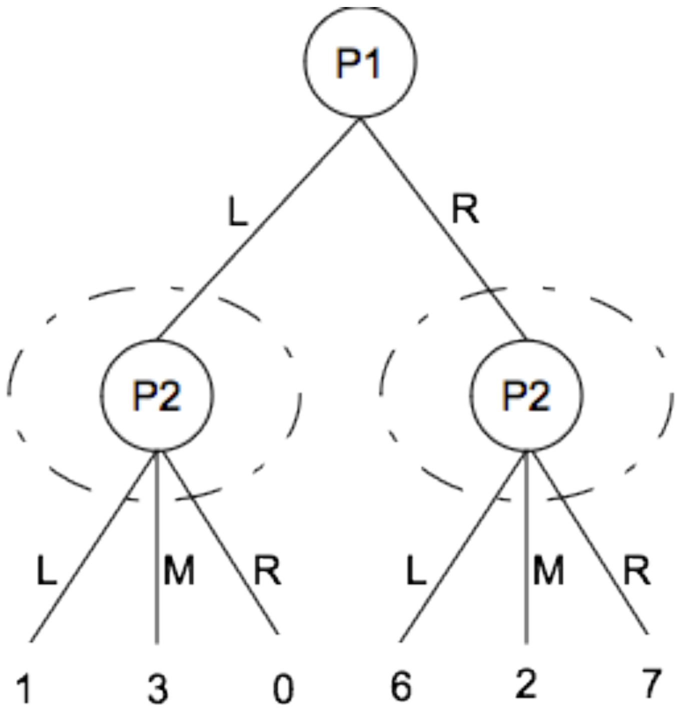
Extensive Games



	L	M	R
L	1	2	0
R	6	2	7

- P1 minimizes and P2 maximizes.
- Mixed strategy solution is {(2/3, 1/3), (1/3, 2/3, 0)}.

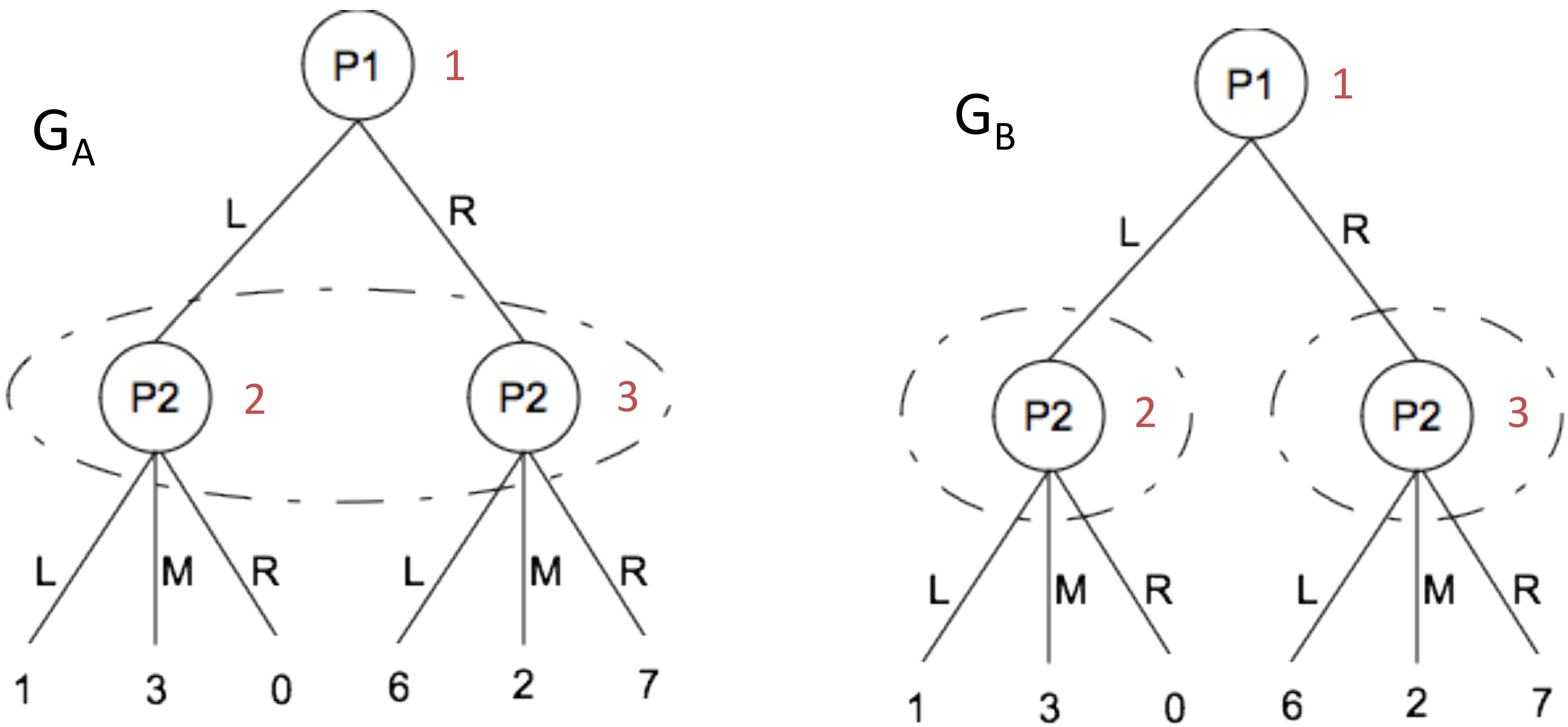
Extensive Games



- P2 has information at the time of his play about what move P1 has made.
- We obtain two pure Nash equilibria (L, MR), (L, ML).
- (L, MR) can be obtained by backward induction.

	u_1	LL	RR	MM	LM	RL	MR	ML	RM
L	1	1	0	3	1	0	3	3	0
R	7	6	7	2	2	6	7	6	2

Information Matters



- G_A: $\eta_1 := \{1\}$, $\eta_2 := \{2,3\}$ (~Open-loop)
- G_B: $\eta_1 := \{1\}$, $\eta_2 := \{\{2\},\{3\}\}$ (~Feedback)

Differential Game

- Players: $\mathcal{N} = \{1, 2, \ldots, N\}$
  - Decision/action for Player i: $u_i \in U_i$.
  - Possible coupled constraints:

    System state x evolves according to the differential equation

$$\dot{x}(t) = f(x(t), u_1(t), \cdots, u_N(t), t), \quad x(0) = x_0$$

  - Each Player i seeks to minimize

$$J_i(u) = \int_0^T \underbrace{F_i(x(t), u_1(t), \cdots, u_N(t), t)}_{\text{Instantaneous Cost}} dt + \underbrace{S_i(x(T))}_{\text{Terminal Value}}$$

Information Structures

- Let $\gamma_i \in \Gamma_i^{\eta}$ be the strategies/policies for the players under information structure η.
- Information structure η can be
  - Open-loop (OL): $u_i(t) = \gamma_i\,(t; x_0)$
  - Feed-back (FB): $u_i(t) = \gamma_i\,(t; x(t))$
  - Closed-loop (CL): $u_i(t) = \gamma_i\,(t; x_{[0,t]})$
  - ε-delayed closed-loop (εDCL):

    $u_i(t) = \gamma_i\,(t; x_{[0,t-\varepsilon]})$, for $t > \varepsilon$; $u_i(t) = \gamma_i\,(t; x_0)$, for $0 \le t \le \varepsilon$.
- Assumptions:
  - γ_i is Lipschitz in x.
  - f is Lipschitz in x and $\{u_1, u_2, \ldots, u_N\}$ are jointly piecewise continuous.

Differential game: Player i solves optimal control problem:

$$(\mathrm{OC}(i))\ \min_{\gamma_i \in \Gamma_i^{\eta *}} J_i(\gamma_i, \gamma_{-i}^{\eta}) := \int_0^T F_i(x, \gamma_i(\eta), \gamma_{-i}^{\eta *}(\eta), t)dt + S_i(x(T))$$

$$\text{s.t. } \dot{x}(t) = f(x, \gamma_i(\eta), \gamma_{-i}^{\eta *}(\eta), t)\,, \quad x(0) = x_0\,.$$

$$J_\mu^{\eta *} = \textstyle\sum_{i \in \mathcal{N}} \mu_i J_i^{\eta *}$$

$$J_\mu^{\eta \circ} = \textstyle\sum_{i \in \mathcal{N}} \mu_i J_i^{\eta \circ}$$

$$\rho_{N,\mu,T}^{\eta} := \max_{\gamma^{\eta *} \in \Gamma^{\eta *}} \frac{J_\mu^{\eta *}}{J_\mu^{\circ}}$$

$$(\mathrm{COC})\ \min_{\gamma \in \Gamma} \sum_{i=1}^{N} \mu_i \left\{ \int_0^T F_i(x(t), \gamma(\eta), t)dt + S_i(x(T)) \right\}.$$

$$\text{s.t. } \dot{x}(t) = f(x, \gamma(\eta), t)\,, \quad x(0) = x_0\,,$$

Team problem under centralized control

Example: Scalar LQ Differential Games

- Each player i minimizes the cost functional:

$$J_i = \int_0^{\infty} \left(q_i x^2(t) + r_i u_i^2(t)\right) dt, \ i \in \mathcal{N}$$

- State dynamics:

$$\dot{x}(t) = a x(t) + \sum_{i=1}^{N} b_i u_i(t), \ x(0) = x_0$$

- $b_i \neq 0$, $r_i \geq 0$, $q_i \geq 0$.
- The feedback Nash equilibrium strategies are linear in state and involve solving a coupled set of algebraic Riccati equations

$$2\left(a - \sum_{i=1}^{N} s_i k_i\right) k_i + q_i + s_i k_i^2 = 0, \ \ \gamma_i^*(x) = -\frac{b_i}{r_i} k_i x, \ i \in \mathcal{N}$$

Comments

- Each player i minimizes the cost functional:

$$J_i = \int_0^\infty \left(q_i x^2(t) + r_i u_i^2(t)\right) dt, \; i \in \mathcal{N}$$

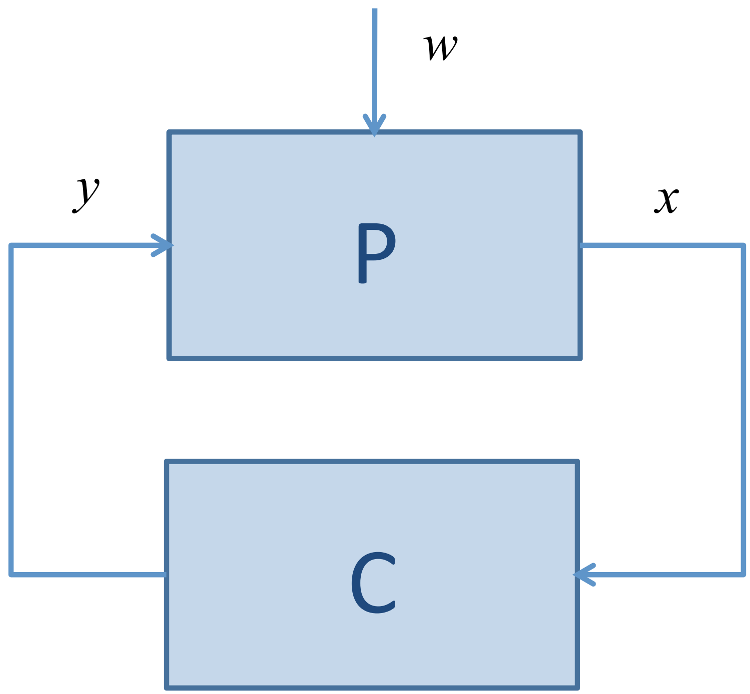
- State dynamics:

$$\dot{x}(t) = ax(t) + \sum_{i=1}^{N} b_i u_i(t), \; x(0) = x_0$$

- Non-uniqueness
  - Informational non-uniqueness
  - Structural non-uniqueness
  - Equilibrium selection (e.g. strong/weak time consistency, robustness to vanishing perturbations)

- Computational complexity
- Large population approximation

Example: H^∞- Optimal Control (Perfect State)



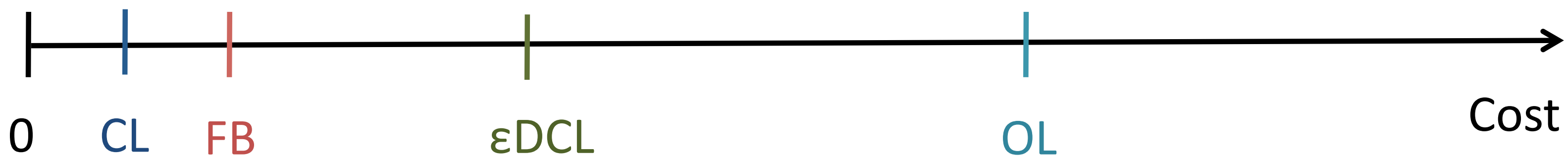
- Plant dynamics: $dx/dt = Ax+Bu+Dw$, $x(0) = 0$, $u = \mu(\eta, t)$

- Zero-sum differential game between u and w.

$$J(\mu,w) = |x(t_f)|^2_{Qf} + ||C\,x||^2 + ||u||^2_R - \gamma^2\,||w||^2$$

[Başar and Bernhard, 1995]

Price of Information (PoI)



- Does it hold for games?

- PoI between two information structures η_1 and η_2:

$$\chi_{\eta_1}^{\eta_2}(\mu) = \frac{\max_{\gamma^{\eta_2^*} \in \Gamma^{\eta_2^*}} J_\mu^{\eta_2^*}}{\max_{\gamma^{\eta_1^*} \in \Gamma^{\eta_1^*}} J_\mu^{\eta_1^*}}$$

[Zhu and Başar, ACC 2010]

Less is More: Case Study

- Let $a = 0$. Then, PoI for the scalar LQ DG is bounded as below independent of system parameters b_i , r_i, q_i .

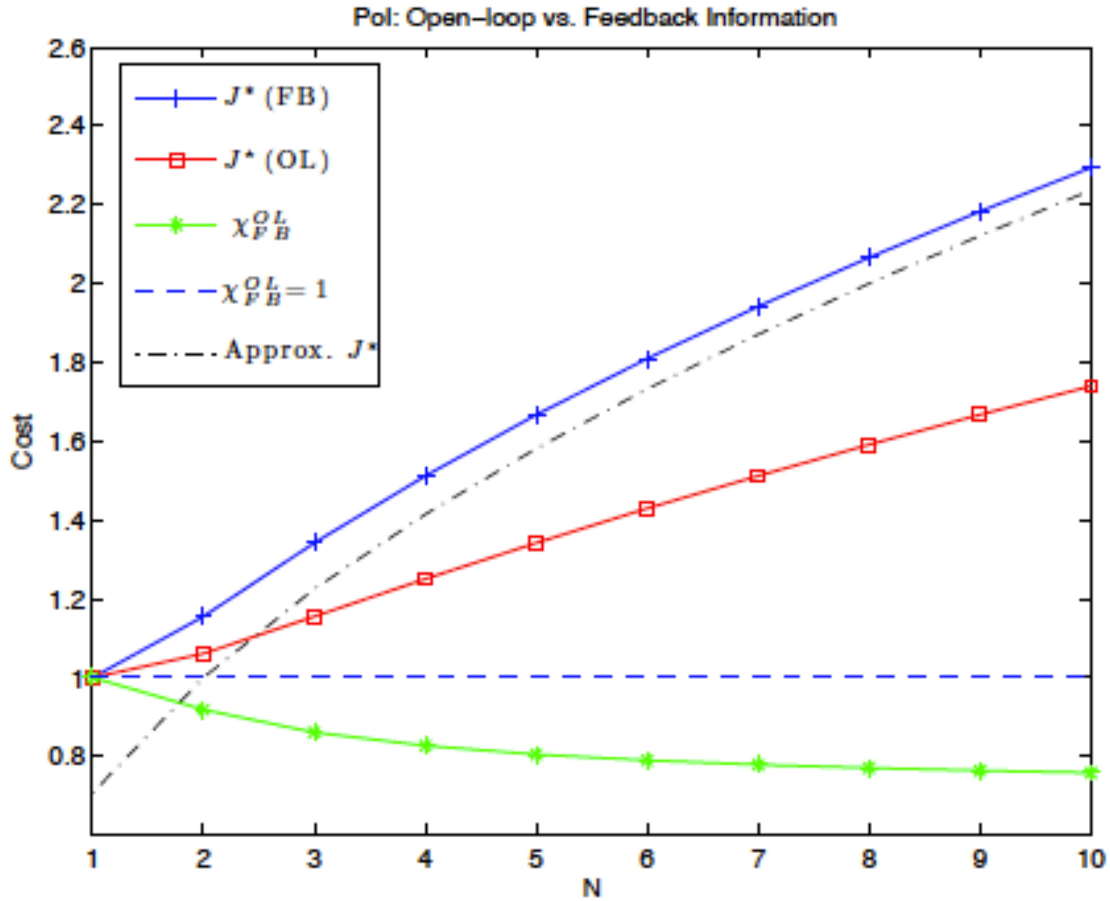
$$\sqrt{2}/2 \leq \chi_{FB}^{OL} \leq \sqrt{2}$$

- Example: multi-user flow control
  - Each user chooses a data rate d_i, equivalently u_i, given service rate.
  - Users needs to circumvent overflow of the queue and minimizes their costs

$$J_i(u) = \int_0^\infty \left(|x(t)|^2 + \frac{1}{c_i} |u_i(t)|^2 \right)$$

$$\dot{q}_l(t) = \sum_{i=1}^{N} u_i(t)$$

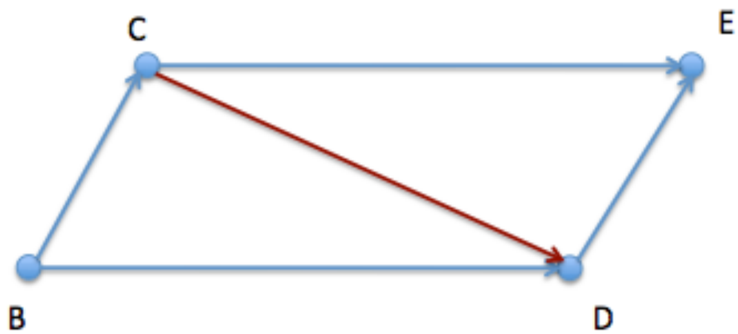
$$u_i(t) := d_i(t) - w_i s_r(t)$$



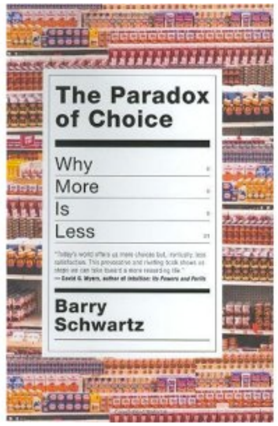
J^* (FB)	J° (TP)	$J^\star$ (OL)	ρ_μ^{FB}	ρ_μ^{OL}	χ_{FB}^{OL}
$\frac{f(N)}{\sqrt{2N-1}}$	$\frac{f(N)}{N}$	$\frac{f(N)}{\sqrt{N}}\left(\frac{1}{2}+\frac{1}{2N}\right)$	$\frac{N}{\sqrt{2N-1}}$	$\sqrt{N}\left(\frac{N+1}{2N}\right)$	$\sqrt{2-\frac{1}{N}\left(\frac{1}{2}+\frac{1}{N}\right)}$

[Başar and Zhu, DGA 2011]

Less is More: A New Paradox?



Braess Paradox



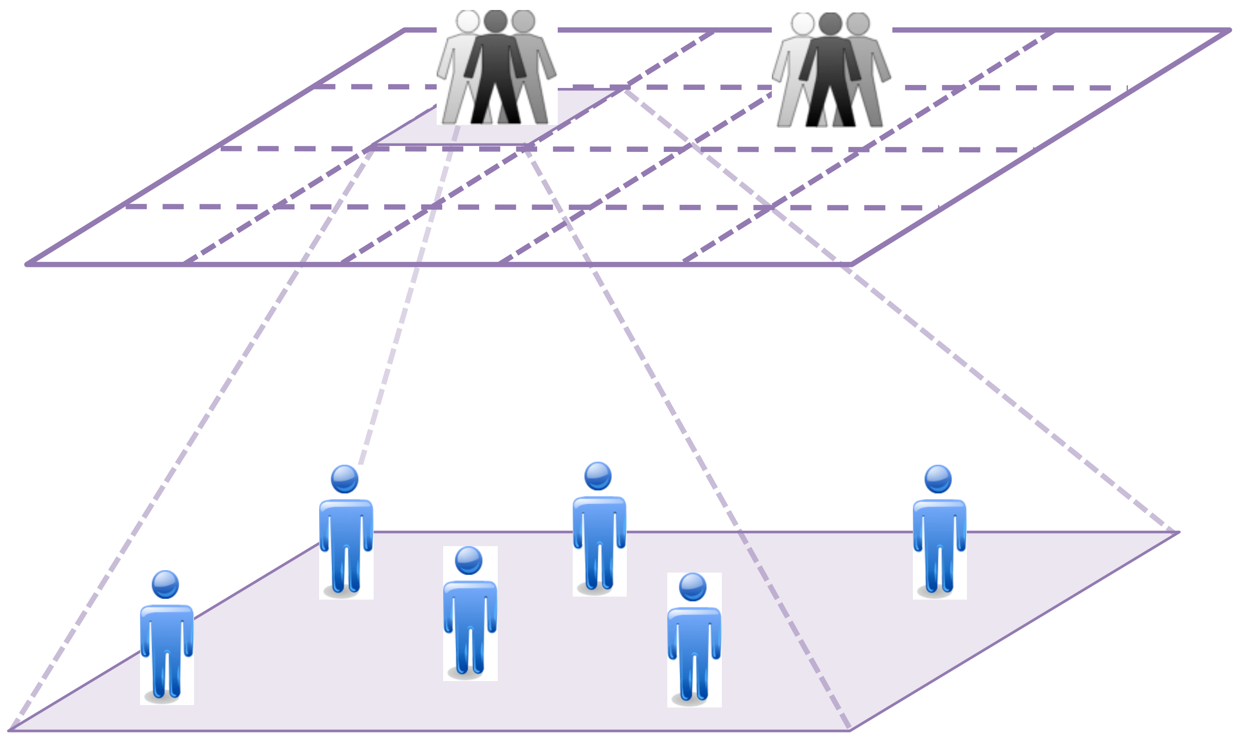
Paradox of Choice

- A paradox of information
  - Players tend to act more conservatively in face of more information.
  - Information is symmetric among players.

Implications

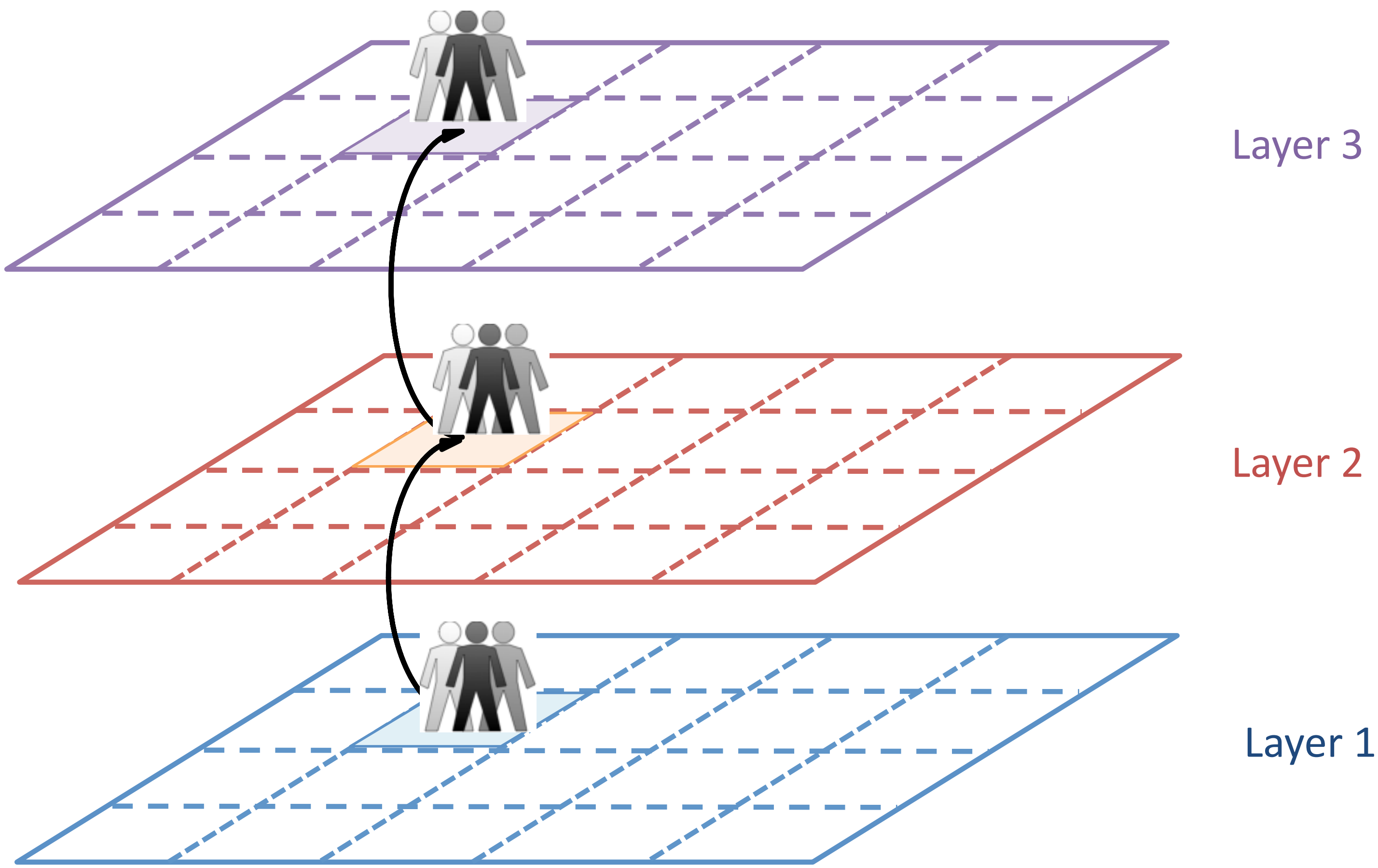
- PoA: Efficiency loss in decentralized architecture in comparison to its centralized counterpart.
- PoI: Information structure implies communication protocol and sensing and monitoring architectures.
- Understanding the tradeoffs between efficiency, robustness, information and resilience.
- Value of Information (VoI):
  - Structural VoI (e.g. PoI, Shapley Value, Indices of Power) × Nonstructural VoI (e.g. Quality, Trustworthiness)

Large Scale Complex Systems



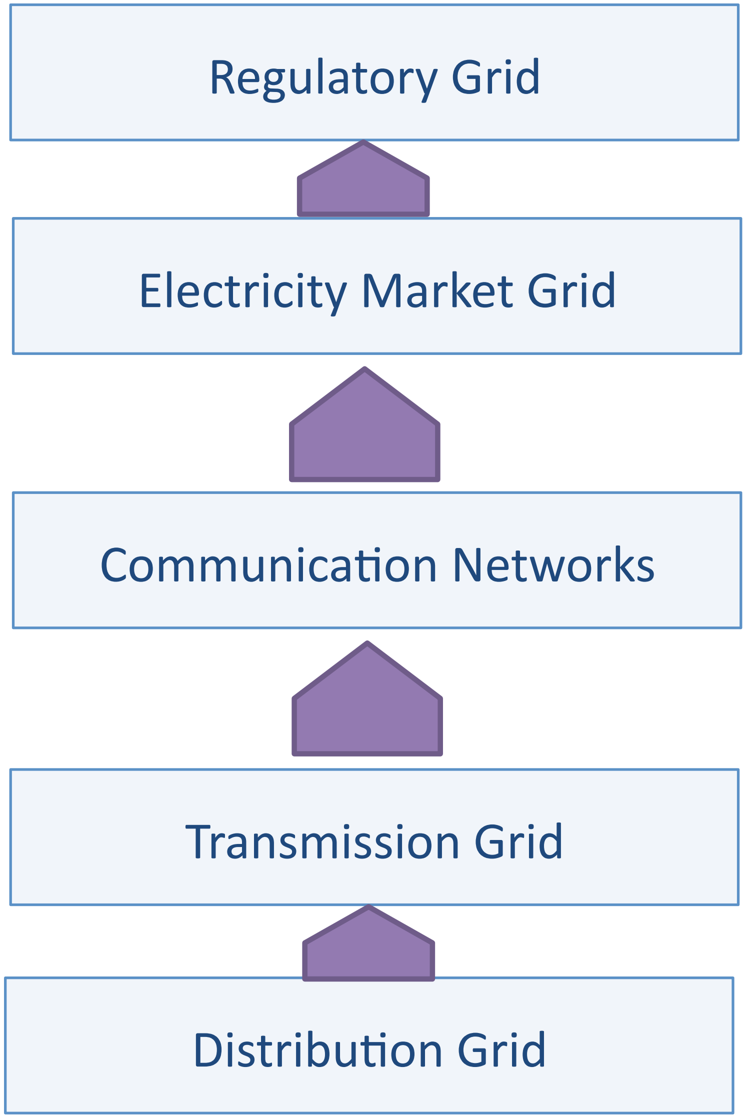
- Interactions happen at different resolutions of the system.
- The number of players is large. The number of groups is large.

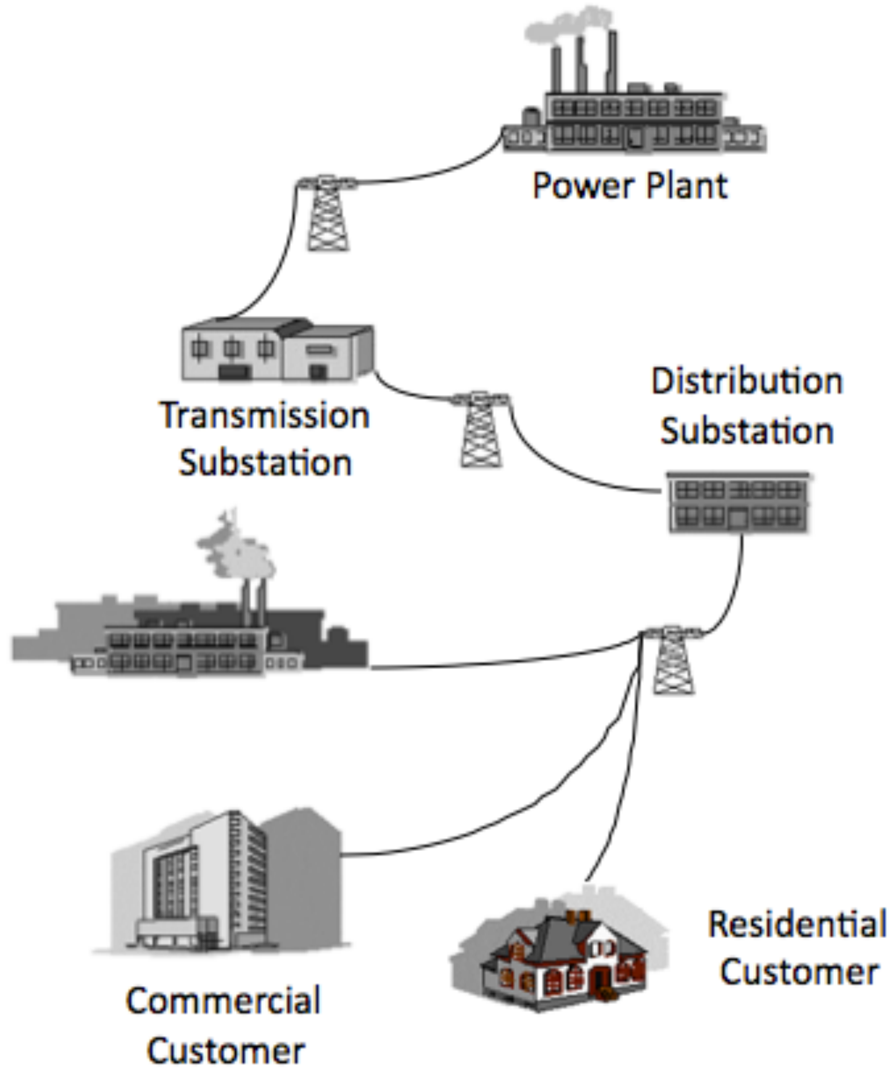
Hierarchical Complex Systems



- Interactions happen at different layers of the complex system.

Power System as a Hierarchical Complex System



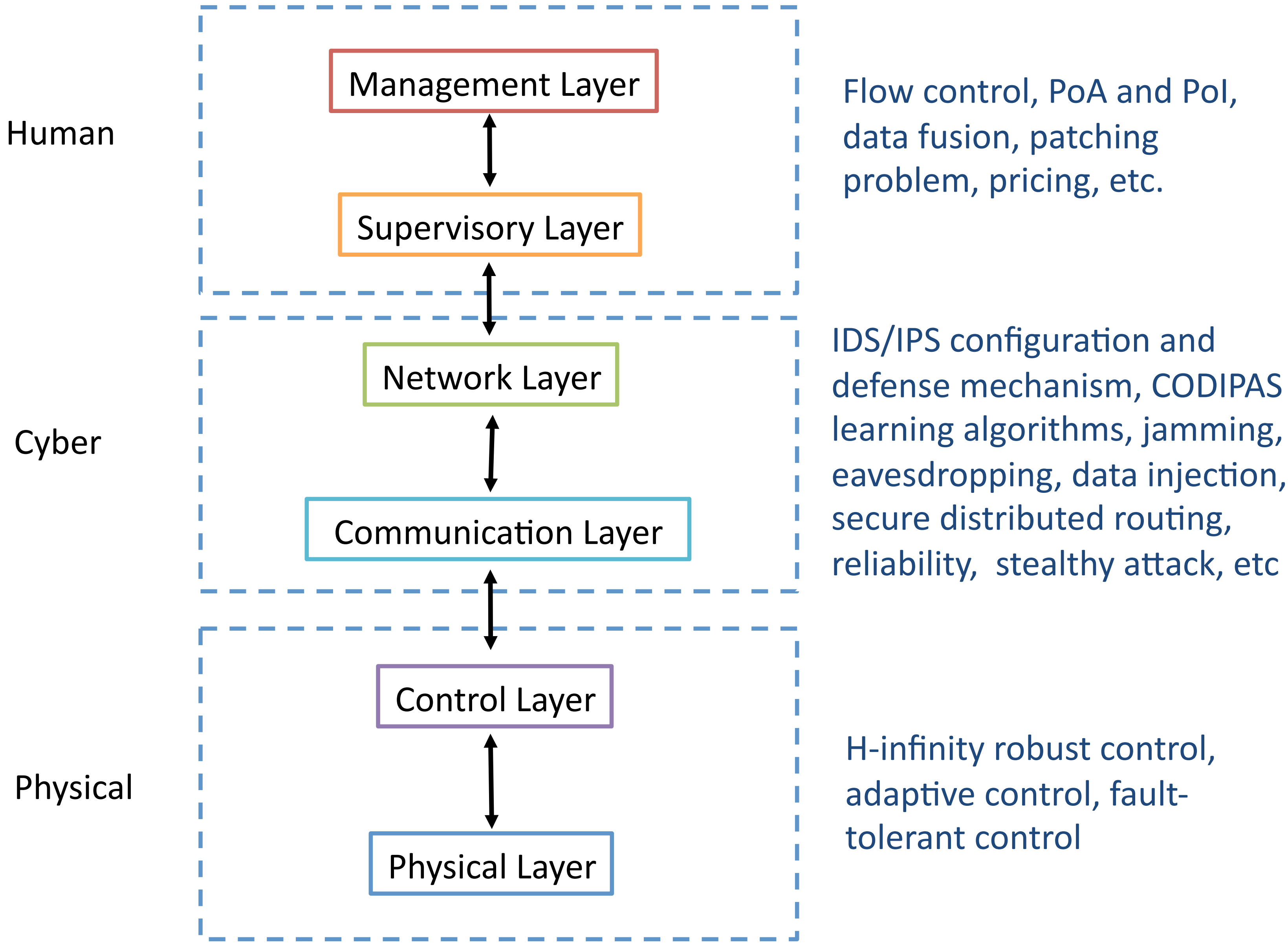


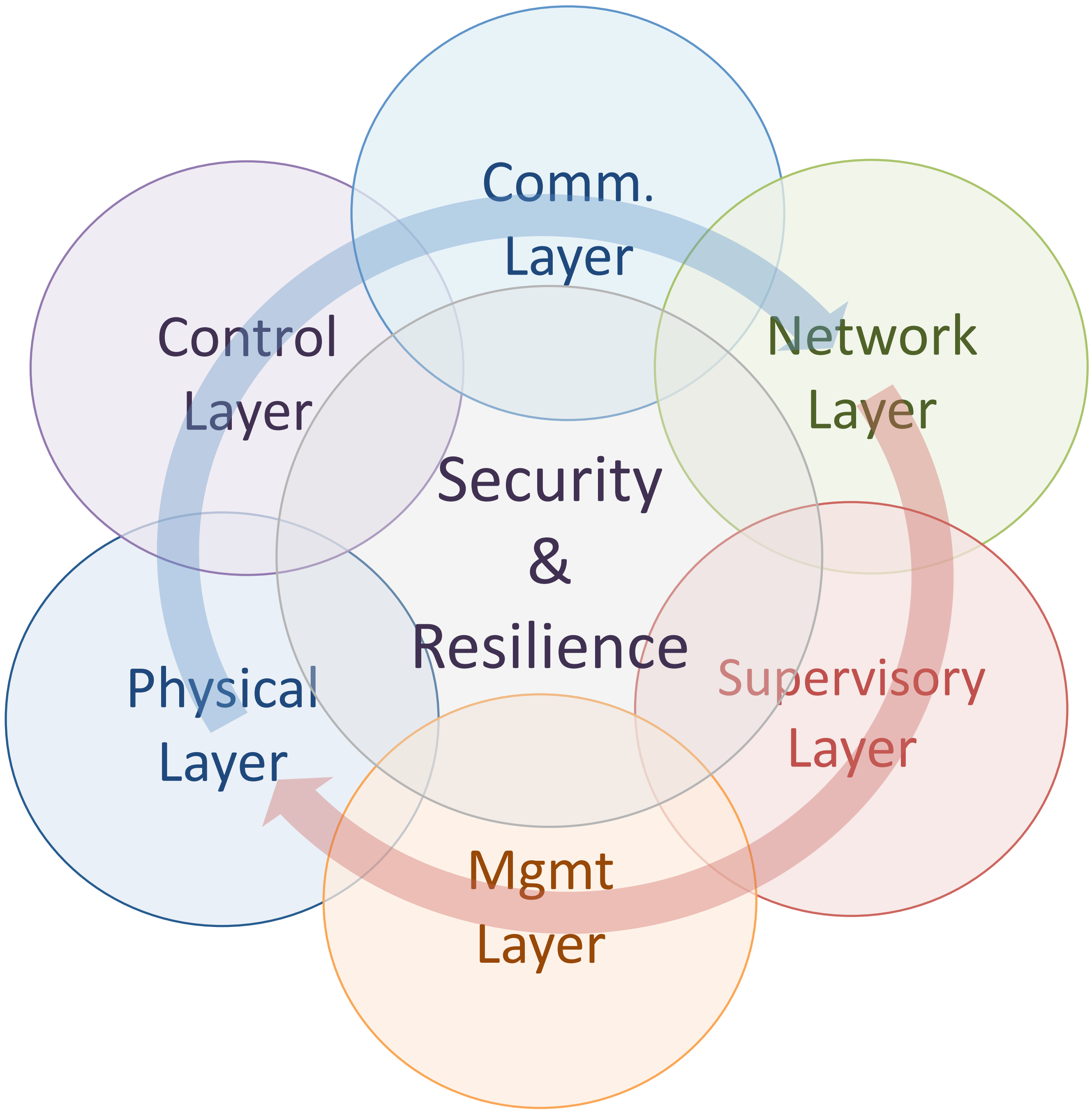
Multi-Resolution (MR) Large Population Games

- MR Stochastic Differential Game Model
- Mean-Field Nash Equilibrium Solution
- Application and Numerical Examples

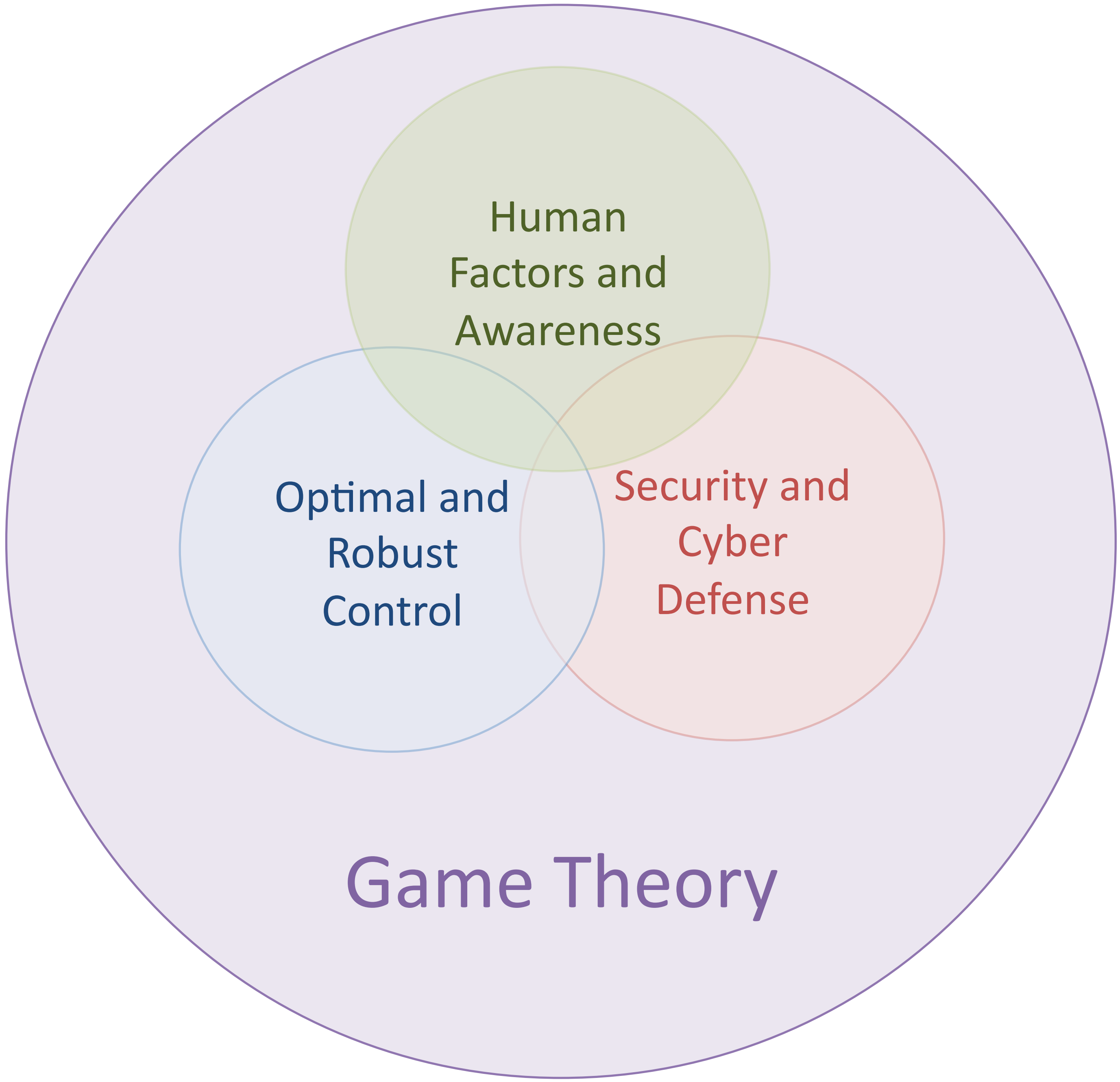
Summary

- Game theory is a versatile tool for analyzing and designing decentralized large-scale systems.
  - Rich literature in economics, mathematics, operations research, computer science, and systems and control.
  - Plenty of room for research on applied side of game theory: mechanism design, learning theory, system theory, decentralized control, etc.
  - Fundamental concepts that enable cross-disciplinary, inter-disciplinary and trans-disciplinary researches.

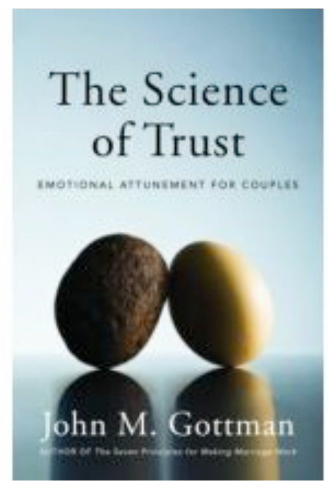

Human
Management Layer
Supervisory Layer
Flow control, PoA and PoI, data fusion, patching problem, pricing, etc.
Cyber
Network Layer
Communication Layer
IDS/IPS configuration and defense mechanism, CODIPAS learning algorithms, jamming, eavesdropping, data injection, secure distributed routing, reliability, stealthy attack, etc
Physical
Control Layer
Physical Layer
H-infinity robust control, adaptive control, fault-tolerant control



Comm.
Layer
Control
Layer
Network
Layer
Security
&
Resilience
Physical
Layer
Supervisory
Layer
Mgmt
Layer


Human
Factors and
Awareness
Optimal and
Robust
Control
Security and
Cyber
Defense
Game Theory

Towards a Science of Security, Resilience and Trust



Agent-based Cyber Control Strategy Design for Resilient Control Systems: Concepts, Architecture and Methodologies

Craig Rieger, Quanyan Zhu and Tamer Başar

Cyber Awareness Track at 11:40 am, August 15