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Toward Global Convergence in Adhoc Networks

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References

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- Some images are courtesy of Internet:
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  - Jie Gao, Stony Brook University, New York
  - Abhishek Tiwari, UtopiaCompression Corporation, Los Angeles, CA
  - Andrew T. Campbell, Dartmouth University, New Hampshire

Data Aggregation

- There are different yet similar definitions:
  - Ability to provide global information for purposes of network management and user services.
  - A set of functions that provide components of a distributed system access to global information.
- Reasons to do Data Aggregation (DA)
  - To coordinate tasks.
  - The need for Node/component duplication for higher performance.
  - The need for redundancy for higher fault tolerance.

Data Aggregation, Cont.

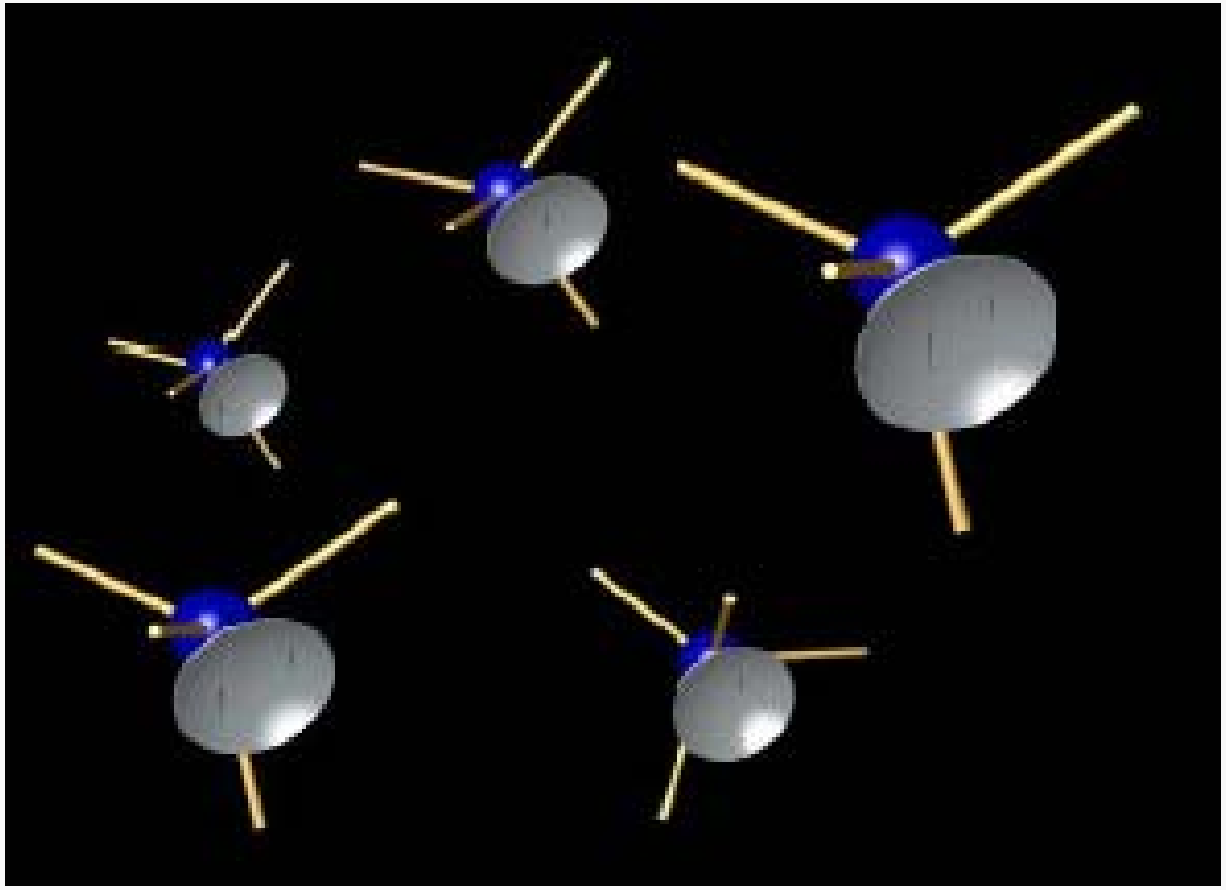
- DA has gone by other names:
  - Data Fusion
  - Approximate Agreement
  - Consensus
  - Distributed Agreement
- Herein these phrases are used interchangeably.

Data Aggregation, Cont.

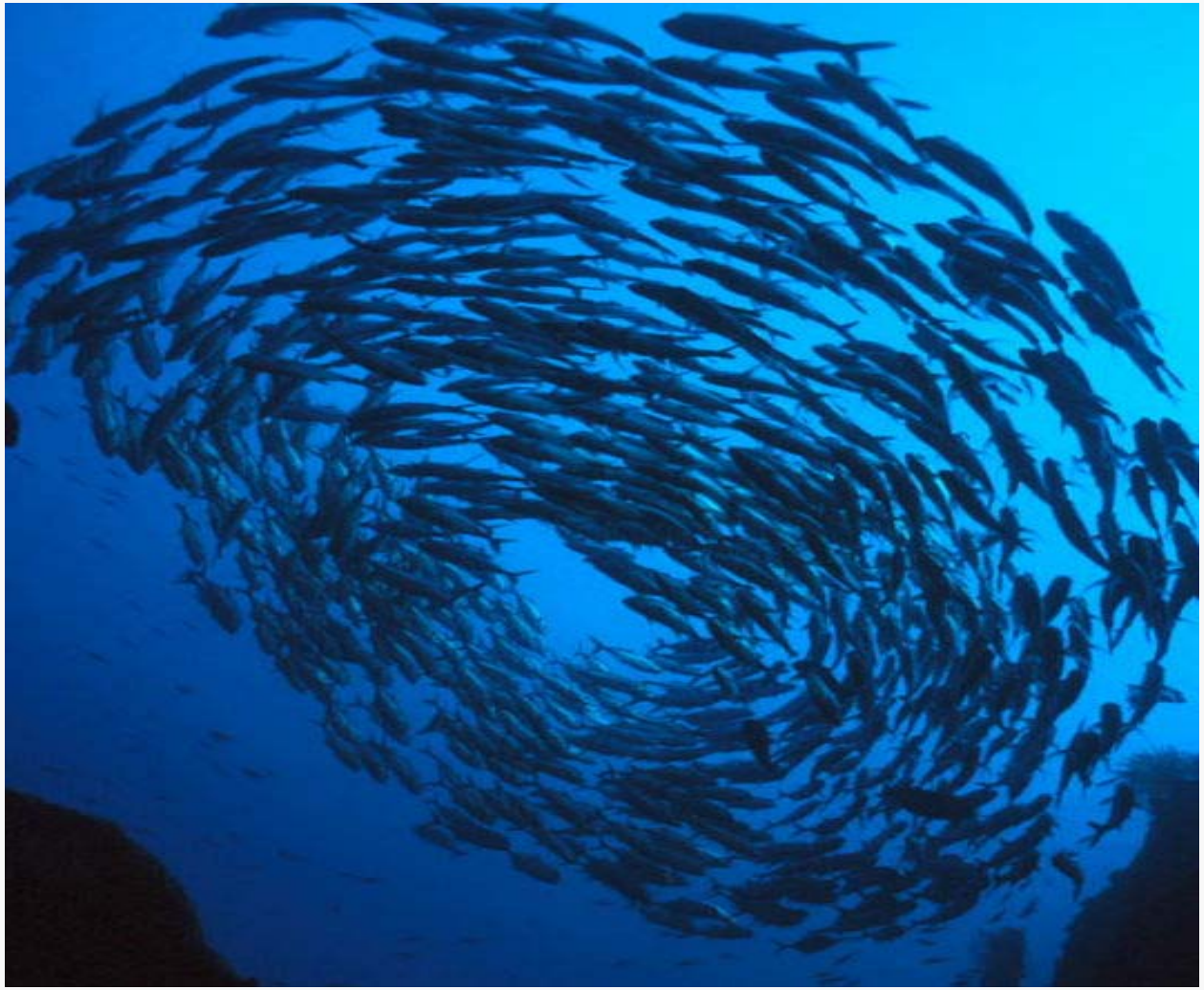
- Some specific examples:
  - Clock synchronization
  - Network size
  - Load balancing
  - Dissimilar (multiversion) software or hardware
  - Incongruent input of numerical data
  - Target trajectory

Some Other/Recent Examples

- Attitude alignment
- Fish swarming
- Bird flocking
- Rendezvous
- Consensus
- Data Fusion

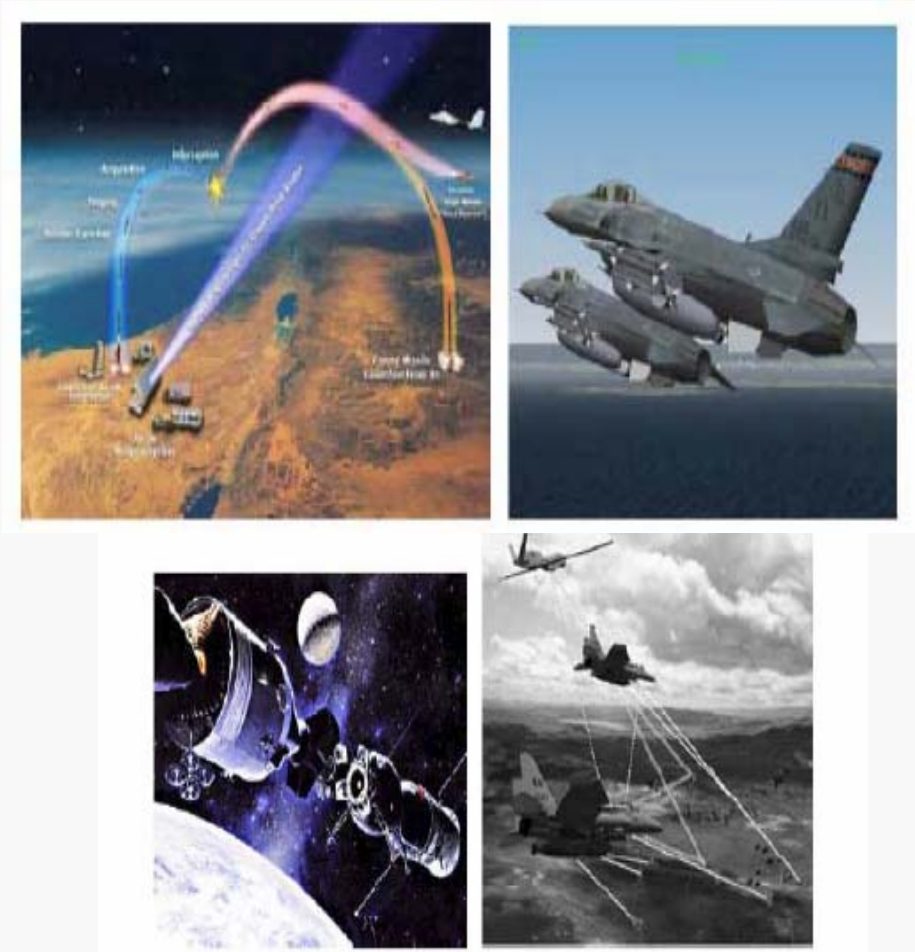


Attitude alignment: among spacecrafts

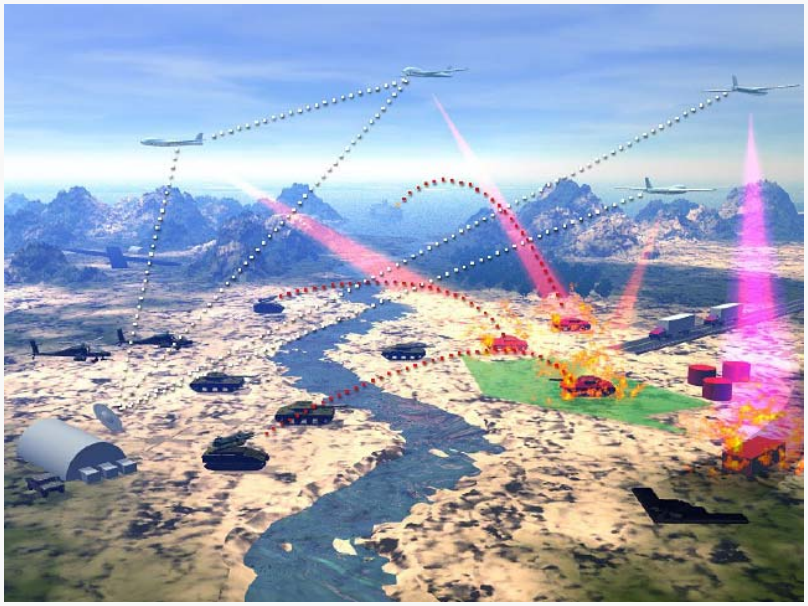


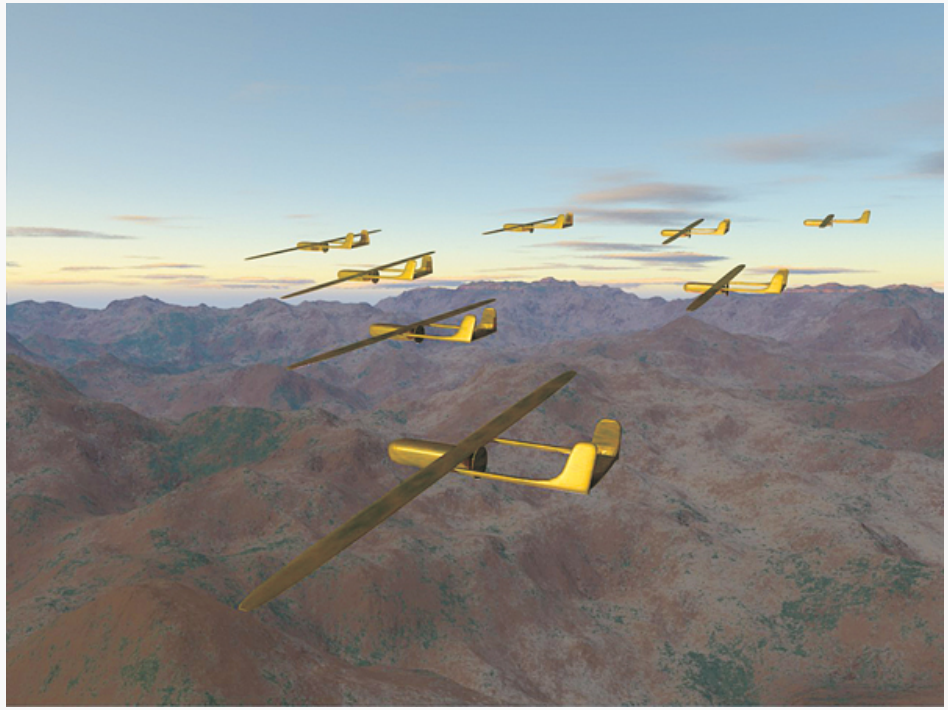
Fish swarming: Cruising together or move in a pattern or direction

Flocking: To move or travel in a group, e.g. in V formation

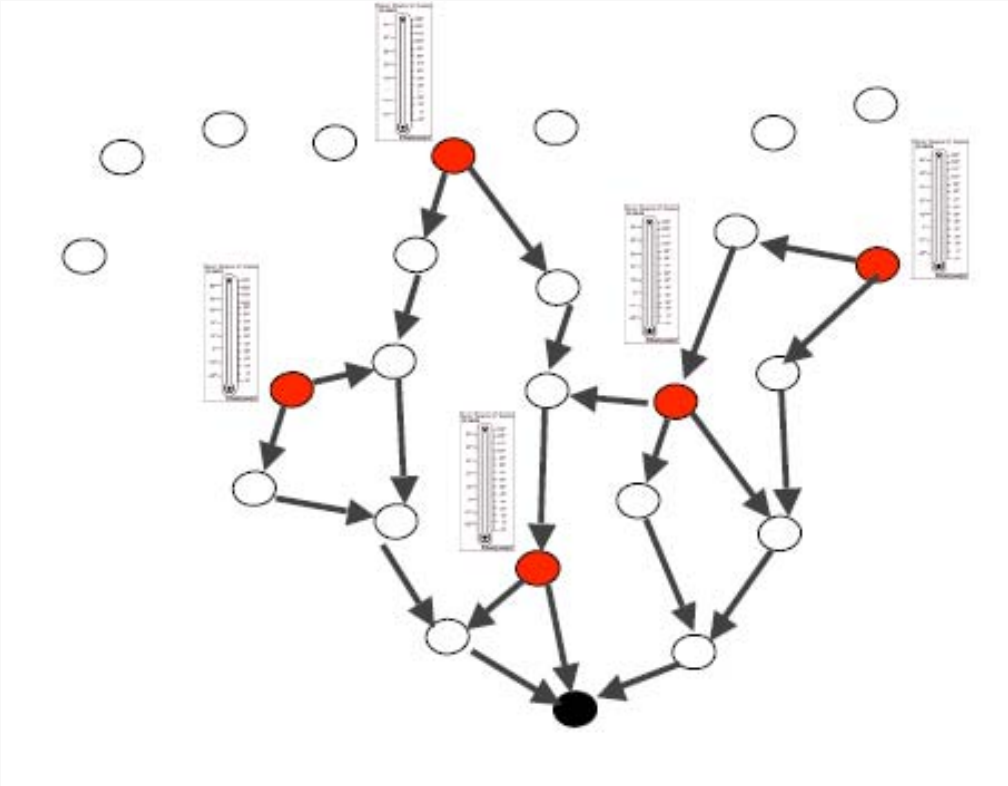


Rendezvous: To reach a desired destination (time or place) within a small time interval of each other





Consensus: Distributed command and control



Data fusion: To collect and diffuse data

What do these examples have in common?

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- Attitude alignment
- Fish swarming
- Bird flocking
- Rendezvous
- Consensus
- Data aggregation

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- Distributed control
- Local / global coordination
- Processes / agents
- Network topology
- Gradual convergence
- Synchrony

Data Aggregation, Cont.

- General Scenario
  - Each node in the network holds a value.
  - Nodes exchange values to decide on values.
- Agreement conditions:
  - **Agreement:** The decided values, one value by each node, are within a predefine tolerance of each other.
  - **Validity:** The decided values are within the range of initial values held by non-faulty nodes.

A Round of Communication

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- Agreement is reached executing multiple rounds.
- Each round consists of:
  1. **Broadcast:** Each node broadcasts its value to others.
  2. **Collect:** Each node forms a multiset of values.
  3. **Filter:** Select values to vote with.
  4. **Average:** Average the selected values to vote.

Two major evolutions

- Static networks
  - Multiprocessor/multiprocessing systems
    - Mostly concentrated on Fully Connected Networks (FCN)
    - Small scale critical systems: power plants, aircrafts, automobiles
  - Distributed Networks
    - Limited focus on Partially Connected Networks (PCN)
    - Large scale distributed systems (Internet)
- Mobile/Adhoc networks
  - Include both FCN and PCN
  - Growing PCN applications: tactical military operations, tracking endangered species, Unmanned Autonomous Vehicles (UAV), etc.

Fully Connected Networks

- Most agreement algorithms are devised for FCNs.
- Full exchange of values in a round is immediate.
- Diameter of values shrinks in each round (single-step).
- Agreement is reached gradually by shrinking diameter in each round.

- Three general approaches:
  - *MSR algorithms*: Remove the extreme-end values.
  - *Egocentric algorithms*: Replace the extreme-end values with your own value.
  - *Egophobic algorithms*: Replace the extreme-end values with your own plus a predefined threshold.

MSR: Remove the extreme-end values

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- Example:
  - Assume 5 nodes with values in the range: 0…1
  - 4 of the nodes posses the values: <0,0,1,1,>
  - The one faulty node (*t=1*) behaves asymmetrically.
  - Use fault tolerant mean function.

  - Case 1: Assume, the faulty node transmits a value outside of the range.

0] 0 1 1 [1.5

0 1 1 → (0+1+1)/3 = 0.66

-1] 0 0 1 [1

0 0 1 → (0+0+1)/3 = 0.33

Average is convergent [(0.66-0.33) < (1-0)] & valid.

MSR: Remove the extreme-end values, Cont.

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- Example, Cont.:
  - Case 2: Assume, the faulty node transmits a value within the range.

$0\,\rfloor\,0 \quad 0.5 \quad 1\,\lfloor\,1$

$0 \quad 0.5 \quad 1 \quad \rightarrow (0+0.5+1)/3=0.5$

$0\,\rfloor\,0 \quad 0.9 \quad 1\,\lfloor\,1$

$0 \quad 0.9 \quad 1 \quad \rightarrow (0+0.9+1)/3=0.63$

Average is convergent [(0.63-0.5) < (1-0)] & valid.

MSR Algorithms

- MSR algorithms can handle the following fault modes:
  - Benign (*b*)
  - Symmetric (*s*)
  - Asymmetric (*a*)
- Thus: $t = a + s + b$

- The voting function:

$$F(\boldsymbol{V}) = mean[Sel_{\sigma}(\mathrm{Re}\,d^{\tau}(\boldsymbol{V}))]$$

where

$$\begin{aligned}
\tau &= (a+s), \text{ maximum number of erroneous values} \\
\mathrm{Re}\,d^{\tau} &= \text{The subsequence } \boldsymbol{M} \text{ remaining after removing} \\
&\quad \text{the } \tau \text{ extreme values} \\
Sel_{\sigma} &= \text{The subsequence } \boldsymbol{S} \text{ of size } \sigma\text{, after selecting } \sigma \\
&\quad \text{elements from } \boldsymbol{M}
\end{aligned}$$

Example

$$
\begin{array}{lcccccccccc}
\boldsymbol{V} = & v_1, & v_2, & v_3, & v_4, & v_5, & v_6, & v_7, & v_8, & v_9, & v_{10} \\
 & & & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\
\boldsymbol{M} = & & & m_1, & m_2, & m_3, & m_4, & m_5, & m_6 & & \\
 & & & \downarrow & & \downarrow & & & \downarrow & & \\
\boldsymbol{S} = & & & s_1, & & s_2, & & & s_3 & & \\
 & & & \downarrow & & \downarrow & & & \downarrow & & \\
 & & & m_{k(1)}, & & m_{k(2)}, & & & m_{k(3)} & &
\end{array}
$$

$$\Rightarrow \quad s_i = m_{k(i)}$$

Some selection functions

- Fault Tolerant Midpoint
- Fault Tolerant Mean
- Fault Tolerant Optimal
- Mixed Mode Optimal
- Binary MSR
- Spanning MSR

MSR Performance for FCN

$$C = \frac{\gamma_a}{\sigma}$$

$\gamma_a \equiv$ the minimum distance between any two elements of $\boldsymbol{S}$ which guarantees that the distance between th e correspond ing elements of $\boldsymbol{M}$ is at least a.

$$N \geq 3a + 2s + b + 1$$

$$V = N - b$$

$$\tau \geq a + s$$

- Partially Connected Networks (PCN)
  - Agreement is not single-step.
  - Number of exchanges (rounds) to reach agreement depends on distribution of values and density of connectivities.
  - Each node is aware of its immediate neighbors only.
  - Most realistic networks are partially connected.

- Research is not matured yet.
  - No unified approach to convergence time/rate
  - No consideration of hybrid fault model.
  - Sporadic research about the criteria on topology affecting convergence time/rate.

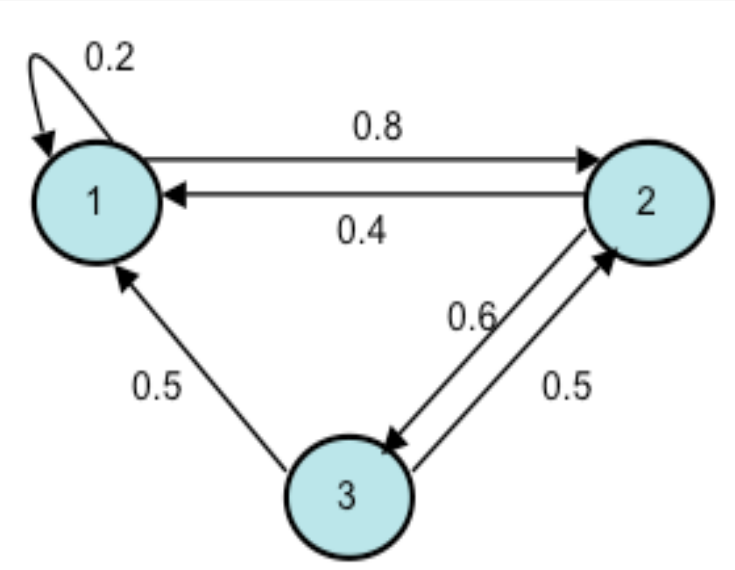
- Objectives:
  - To achieve AA conditions in a PCN.
  - To achieve the conditions needed for a PCN to reach AA conditions.
  - To show the number of rounds needed to reach a single network-convergence.
  - To obtain the asymptotic stationary-convergence.
  - To form a basis for future research.

- Approach:
  - Exploit properties of Markov Models (Markov Chains).
  - Exploit convergence in a fixed adhoc network.
  - Exploit convergence in a fixed adhoc network in the presence of omission faults.

- Eigenvector and eigenvalue:
  - x is an eigenvector of a matrix M if: $Mx = \lambda x$.
  - λ is a scalar value and is the eigenvalue corresponding to the eigenvector x.
  - A matrix of size $n \times n$ has n eigenvalues.
  - In a stochastic matrix with non-zero diagonal values, for each λ:

$$-1 < \lambda <= 1.$$

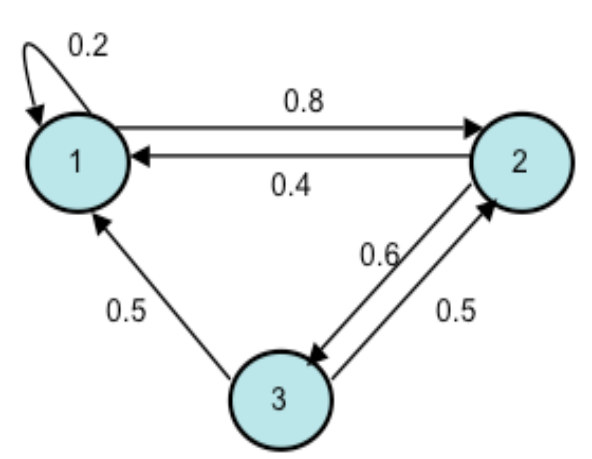
Markov Chain Example



$$\prod^{0} = [0.1, 0.7, 0.2]$$

$$P = \begin{bmatrix} 0.2 & 0.8 & 0 \\ 0.4 & 0 & 0.6 \\ 0.5 & 0.5 & 0 \end{bmatrix}$$

- Probability of being in the next state:



$$\prod^{1} = \prod^{0} P = [0.1, 0.7, 0.2] \begin{bmatrix} 0.2 & 0.8 & 0 \\ 0.4 & 0 & 0.6 \\ 0.5 & 0.5 & 0 \end{bmatrix} = [0.4, 0.18, 0.42]$$

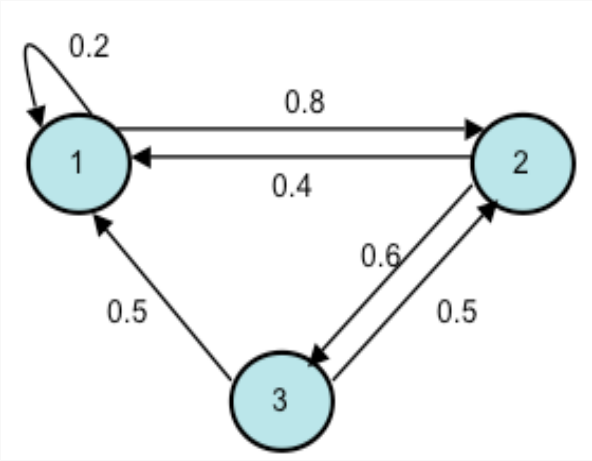
- Probability of being in a state in two steps:

$$\prod^2 = \prod^1 P = \prod^0 PP = \prod^0 P^2$$

- Probability of being in a state in *s* steps:

$$\prod^s = \prod^0 P^s$$

- For this example:



$$P^2 = \begin{bmatrix} 0.36 & 0.16 & 0.48 \\ 0.38 & 0.62 & 0.00 \\ 0.30 & 0.40 & 0.30 \end{bmatrix} \qquad P^6 = \begin{bmatrix} 0.35 & 0.40 & 0.26 \\ 0.36 & 0.41 & 0.23 \\ 0.35 & 0.41 & 0.24 \end{bmatrix}$$

$$P^8 = \begin{bmatrix} 0.35 & 0.40 & 0.24 \\ 0.35 & 0.40 & 0.24 \\ 0.35 & 0.40 & 0.24 \end{bmatrix} \qquad P^9 = \begin{bmatrix} 0.35 & 0.40 & 0.24 \\ 0.35 & 0.40 & 0.24 \\ 0.35 & 0.40 & 0.24 \end{bmatrix}$$

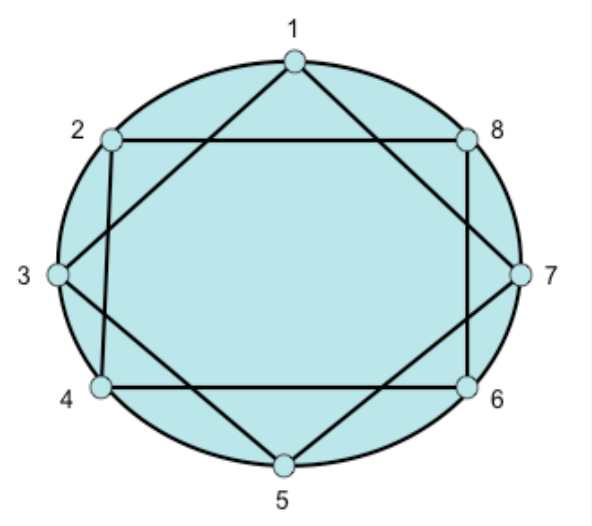
Convergence Characteristics of Transition Matrix

- The sum of every row is 1.
- A transition matrix is stable when multiplying it by itself produces the same matrix.
  - In a stable matrix all rows are the same.
- A transition matrix of a corresponding Markov Chain when raised sufficient number of times becomes stable if:
  - The Markov Chain is irreducible.
  - The Markov Chain is aperiodic.

Network Model Assumptions

- Network is partially connected.
- Network is strongly connected.
- Each node is capable of broadcasting its value to its neighbors.
- Communication is synchronous.
- Links are bidirectional.

- Example: Chordal Ring



$$V_0 = [1,2,3,4,5,6,7,8]$$

$$A = \begin{bmatrix} 1/5 & 1/5 & 1/5 & 0 & 0 & 0 & 1/5 & 1/5 \\ 1/5 & 1/5 & 1/5 & 1/5 & 0 & 0 & 0 & 1/5 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 & 0 & 0 & 0 \\ 0 & 1/5 & 1/5 & 1/5 & 1/5 & 1/5 & 0 & 0 \\ 0 & 0 & 1/5 & 1/5 & 1/5 & 1/5 & 1/5 & 0 \\ 0 & 0 & 0 & 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/5 & 0 & 0 & 0 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/5 & 1/5 & 0 & 0 & 0 & 1/5 & 1/5 & 1/5 \end{bmatrix}$$

- The voted values after one round of exchange:

$$V_1 = V_0 A$$

- The voted values after two rounds of voting:

$$V_2 = V_0 A^2$$

- After 15 rounds of voting:

$$V_{15} = V_0 A^{15}$$

Chordal Ring, Cont.

$$A^2 = \begin{bmatrix} 0.20 & 0.16 & 0.12 & 0.08 & 0.08 & 0.08 & 0.12 & 0.16 \\ 0.16 & 0.20 & 0.16 & 0.12 & 0.08 & 0.08 & 0.08 & 0.12 \\ 0.12 & 0.16 & 0.20 & 0.16 & 0.12 & 0.08 & 0.08 & 0.08 \\ 0.08 & 0.12 & 0.16 & 0.20 & 0.16 & 0.12 & 0.08 & 0.08 \\ 0.08 & 0.08 & 0.12 & 0.16 & 0.20 & 0.16 & 0.12 & 0.08 \\ 0.08 & 0.08 & 0.08 & 0.12 & 0.16 & 0.20 & 0.16 & 0.12 \\ 0.12 & 0.08 & 0.08 & 0.08 & 0.12 & 0.16 & 0.20 & 0.16 \\ 0.16 & 0.12 & 0.08 & 0.08 & 0.08 & 0.12 & 0.16 & 0.20 \end{bmatrix}$$

$$V_2 = V_0 A^2 = [4.20, 3.92, 3.96, 4.32, 4.68, 5.05, 5.08, 4.80]$$

$$A^{15} = \begin{bmatrix} 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 \\ 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 \\ 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 \\ 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 \\ 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 \\ 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 \\ 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 \\ 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 \end{bmatrix}$$

$$V_{15} = V_0 A^{15} = [4.5, 4.5, 4.5, 4.5, 4.5, 4.5, 4.5, 4.5]$$

Convergence in PCN

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- What does convergence mean in PCN?
  - When diameter of values shrinks in a round.
- Convergence occurs when either of following occurs:
  - Either of extreme-end values disappears in a round.
  - Both extreme-end values disappear in a round.

Convergence in PCN, Cont.

- Convergence takes place in phases:
  - *Phase*: A number of rounds in which a convergence occurs.
- Multiple phases are needed for stationary-convergence.
  - Each phase causes a shrink in diameter.
  - Multiple phases cause *agreement* within ε.

Convergence in PCN, Cont.

- Number of rounds in a phase is reduced when:
  - Number of extreme-end values is low.
  - Extreme-end and none extreme-end values are neighbored.
- Number of rounds in a phase is increased when:
  - Number of extreme-end values is high.
  - Extreme-end values in each set are packed together.
- Number of rounds to convergence is highest when:
  - Number of both extreme-end values are highest and equal to each other (or almost equal).
  - Extreme-end values in each set are packed tightly.

- Number of rounds in a phase can not be greater than the diameter m of the network.
  - The effect of a value held by a node reaches the farthest node due to A^m.
  - Of course, this occurs if the network is strongly connected.

- More specifically, the maximum number of rounds in a phase for strongly connected, aperiodic network is:

$$R(m) = \left\lceil \frac{m}{2} \right\rceil$$

- Proof is by induction on the number of nodes N in the network.

Number of rounds to stationary-convergence

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- The number of rounds to reach the stationary-convergence is:

$$\log_{\lambda} \varepsilon + r - 1$$

- ε is a small predefined tolerance value.
  - It is an indicator of how close the voted values are to each other.
- r is the multiplicity of the second largest eigenvalue.
- λ is the magnitude of the second largest eigenvalue.

Convergence rate

- The average convergence rate per round is:

$$C = \left(\frac{c\,\varepsilon}{\delta(\boldsymbol{U})} \right)^{1/k}$$

- k is the number of rounds to stationary-convergence.
- c is a constant value.
  - It is difficult to obtain, as it depends on the topology.
  - Therefore, a smaller ε provides a better convergence rate.
- Note: in reality, a convergence may not happen in a round.

Considering Omission Faults

- The previous discussion assumed the network topology is the same from one round to another.
- In many applications this may not be a reality:
  - Omissions can be common in wireless networks.
  - Hence, the stochastic matrix can change from one round to another.

Definitions

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$G \equiv$ The original network topology in fault free environment.

$n \equiv$ The number of vertices in G.

$m \equiv$ The diameter of G.

$d \equiv$ The maximum node indegree of G.

$A \equiv$ The adjacency matrix corresponding to G.

$G_i \equiv$ The topology in round i.

$A_i \equiv$ The adjacency matrix corresponding to G_i.

$d_i \equiv$ The indegree of node i in a round.

$D_i \equiv$ The diagonal matrix corresponding to G_i,

where for a node j, $[D_i]_{jj} = d_j$

$\ell \equiv \lceil m/2 \rceil$

- The stochastic matrix in round i is obtained as:

$$M_i = D_i^{-1} A_i$$

Definitions, Cont.

$V_i \equiv$ Multiset of values received in the current round by node i.

$F(V_i) \equiv$ The voting function used by node i.

$\boldsymbol{V}_i \equiv$ The multiset of values in the entire network for round i.

$\boldsymbol{F}(\boldsymbol{V}_i) \equiv$ The voting function applied to the entire network for round i.

- Therefore, the voted values in round *k* are:

$$\boldsymbol{F}(\boldsymbol{V}_k) = \left[F(V_1), F(V_2), \ldots, F(V_n)\right] = M_k \boldsymbol{V}_k = M_k M_{k-1} M_{k-2} \ldots M_1 \boldsymbol{V}_1$$

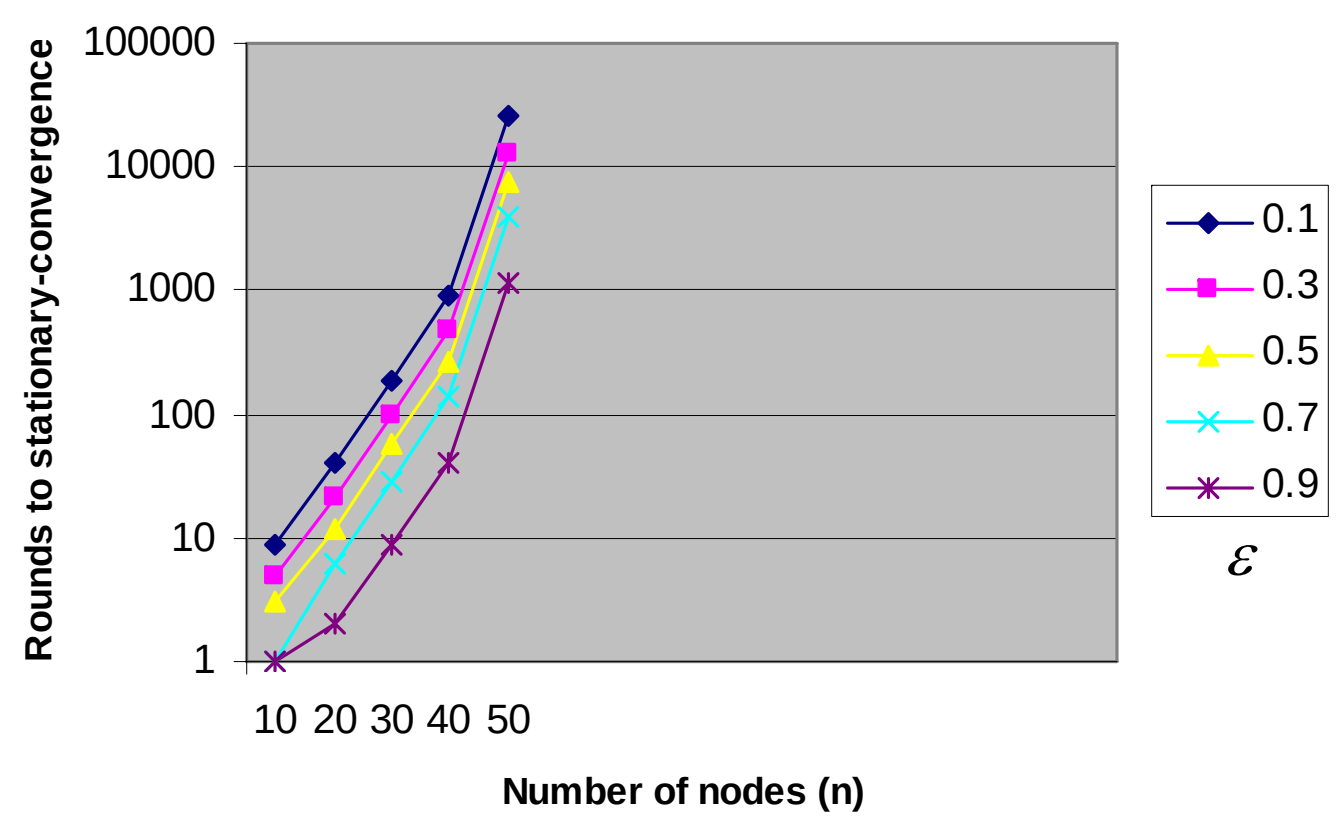
Number of rounds to stationary-convergence

- The upper bound on the number of rounds to achieve the stationary-convergence is:

$$\frac{\ell}{\log_{\varepsilon}(1 - n(1/d)^{\ell})}$$

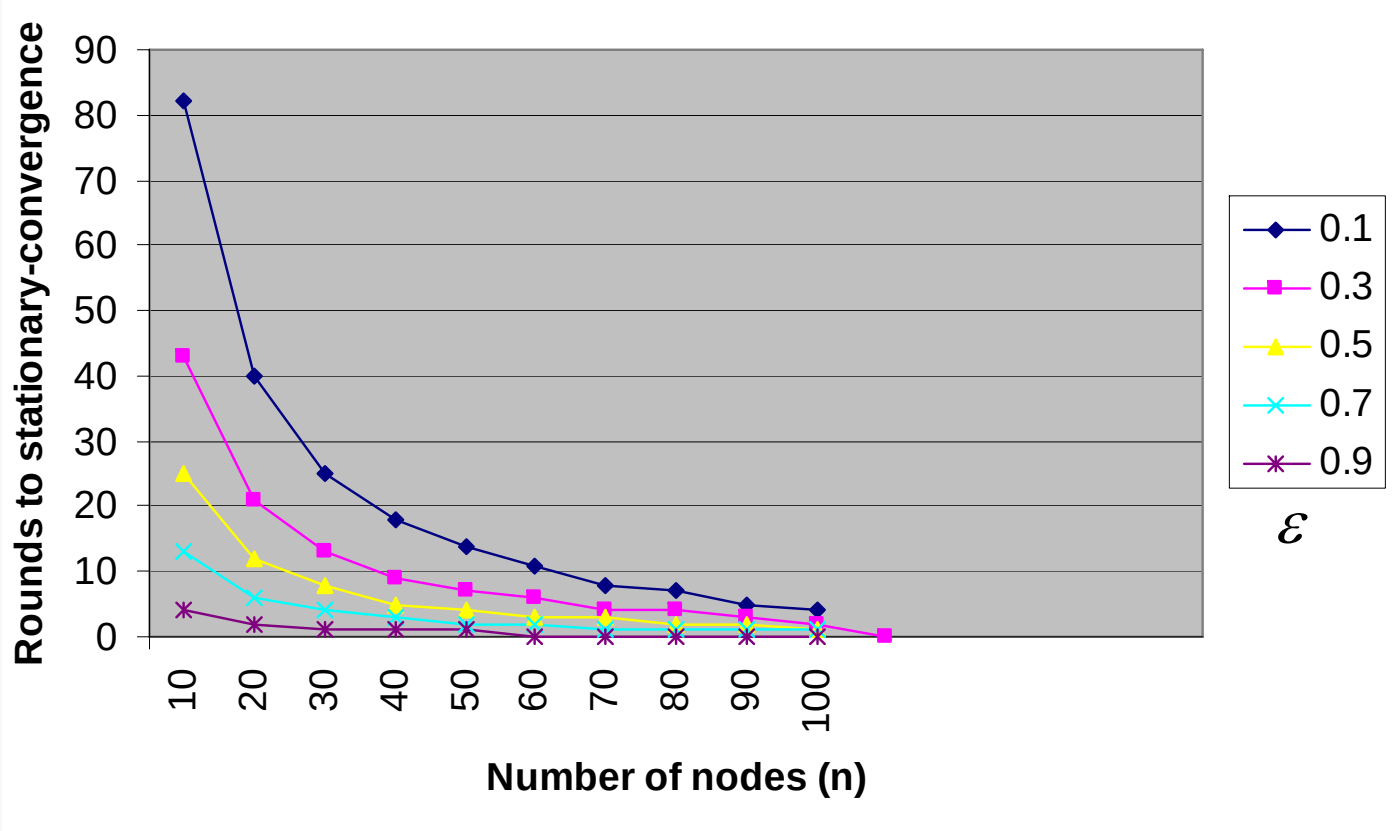
- Number of rounds in chordal rings of 2nd order:

$$m = \lceil n/4 \rceil,\ \ell = \lceil m/2 \rceil,\ d = 5$$



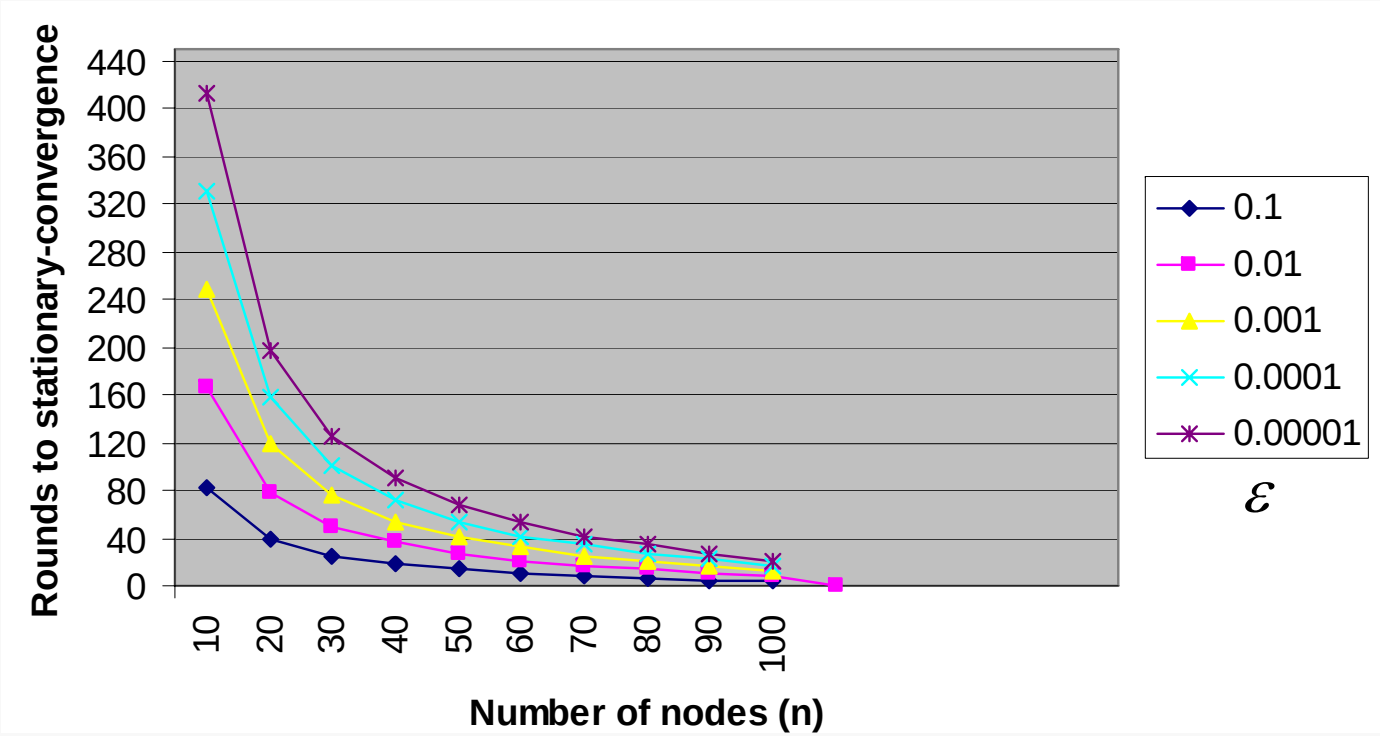
- Number of rounds in a network with:

$$m = 5,\ \ell = 3,\ d = 5$$



- Number of rounds in the same network, but when the precision is increased:

$$m = 5,\ \ell = 3,\ d = 5$$



Remarks

- Network-diameter has a big effect on the number rounds to reach the stationary-convergence.
- The speed of convergence is highly dependent on the distribution of values.
- These examples considered the worst-case, i.e. every phase takes the highest number of rounds.
- If divergence prevention is needed, just a single phase executed at regular times should be sufficient.

Future research

- To place a bound on the convergence rate,
- To consider malicious faults,
- To run simulation results against the theoretical results,
- To consider node mobility.

Questions?

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