ISRCS 2012

Passivity-Based Trajectory Tracking Control with Adaptive Sampling Over a wireless Network

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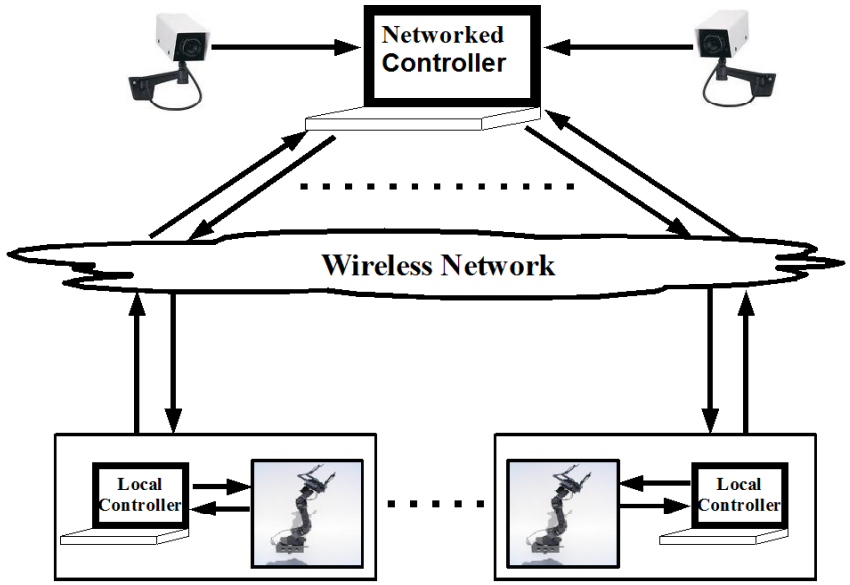
Motivation



Networked Multi-Robot Control System

Challenges

- **Impact of network effects**
  - Time varying delays and packet loss
- **Limited Network and Computational Resources**
  - Efficient utilization of resources (update frequency)
  - Limitations in the achievable stability and performance

Outline

- **Overview of Passivity**
- **Overview of Adaptive Sampling**
- **PBASC Architecture**
- **Results**
- **Case Study**
- **Conclusion**

Passivity

- Abstraction for describing the input-output behavior of a system in terms of intuitive concepts such as power and energy
- Sufficient Condition for Stability
- Passive systems store or dissipate energy, but cannot creating it.
- Wide range of applications
  - Linear/Nonlinear, Discrete/Continuous, Distributed systems
- Definition

$$\text{H:} \begin{cases} \dot{x} = f(x,u) \\ y = g\,(x,u) \end{cases}$$

Passivity: $\int_0^T w(t)dt = \int_0^T u^T y\, dt \geq V(x(T)) - V(x(0))$

Strictly Output Passivity: $\int_0^T w(t)dt = \int_0^T (u^T y - \epsilon y^T y\,)dt \geq V(x(T)) - V(x(0))$

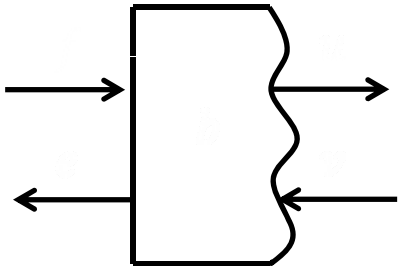
Strictly Input Passivity: $\int_0^T w(t)dt = \int_0^T (u^T y - \delta u^T u)dt \geq V(x(T)) - V(x(0))$

Wave Variables

- **Important tool** used in relation to passivity
  - Exchange of information
- **Scattering Transformation**: Transform modeling transmitted and reflected waves, u and v:

$$u = \frac{1}{\sqrt{2b}}(bf + e)$$

$$v = \frac{1}{\sqrt{2b}}(bf - e)$$

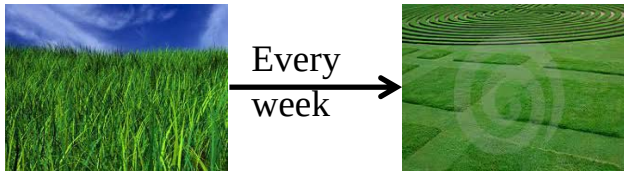


- Extension to passivity theory to deal with time delays
  - Couples power variables (effort e and flow f)
  - Delayed waves preserve energy content
  - b is the *characteristic impedance*
- Preserves power flow P:

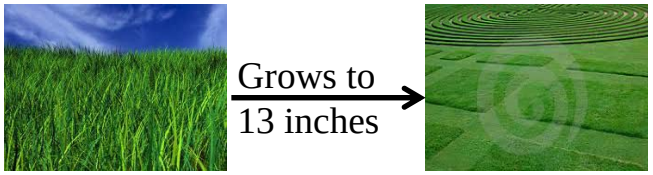
$$P = e^T f = \frac{1}{2} u^T u - \frac{1}{2} v^T v$$

Adaptive Sampling

- **Limited resources**
  - How frequent should one sample?
  - How often should a control law be computed and updated?
- **Classical Approach**
  - Fixed sampling interval (well-established)
  - Open-loop sampling approach



- **Adaptive Sampling**
  - Occurrence of an event triggers control and communication
  - Reduces computation, transmissions and energy consumption



- **Two main Categories**
  - Events from the Control Layer of the NCS
    - Event-Triggered Control and Self-Triggered Control
  - Network-based QoS Sampling (Rate allocation, scheduling, sampling rate adaptation)

Objective

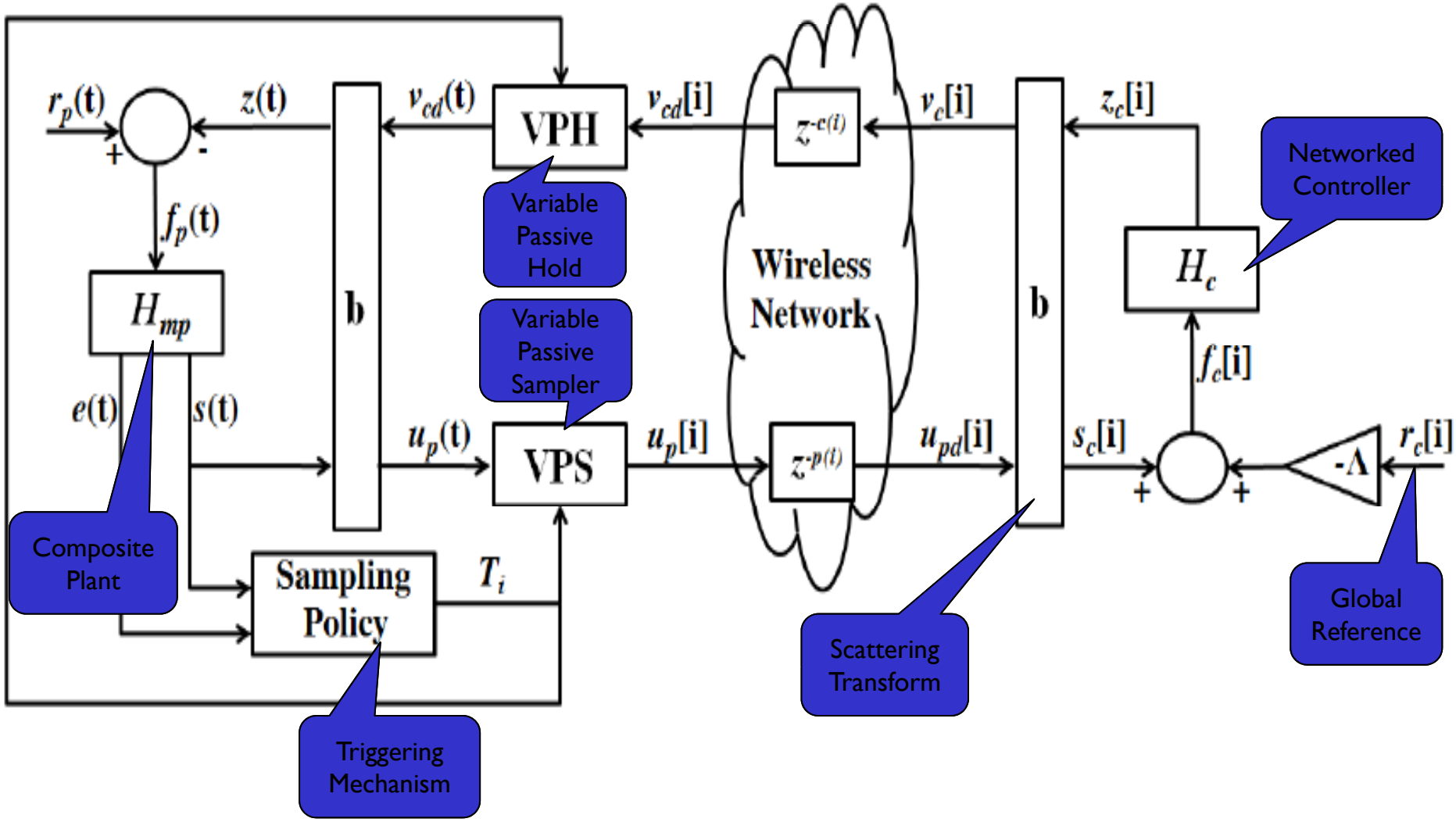
- Trajectory Tracking
  - Efficient use of network resources
  - Unreliable network conditions (packet loss and time delays)
- Trajectory tracking is defined as

$$\lim_{t \to \infty} y_p(t) - y_d(t) = 0$$

Approach

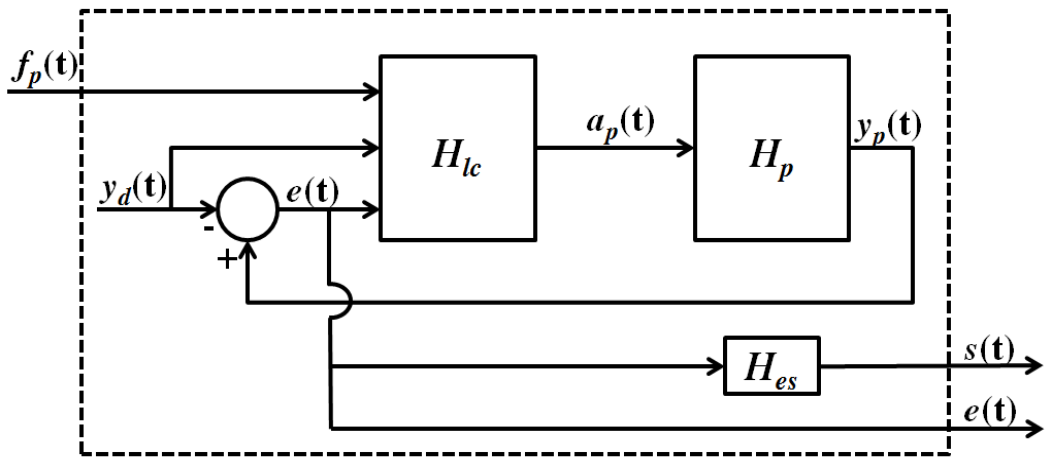
- Integration of two control design concepts
  - Passivity
  - Adaptive Sampling
- Tracking control in terms of stabilization of the system in error space

Passivity-Based Adaptive Sampling Control Architecture



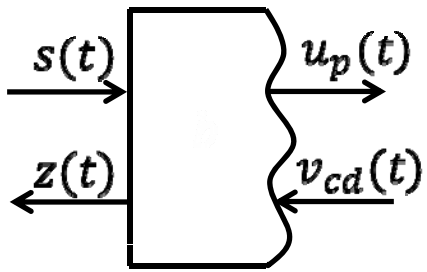
Composite Plant, H_{mp}

- Local Controller, H_{lc}
- Error function, H_{es}: $s = \dot{e} + \Lambda e$



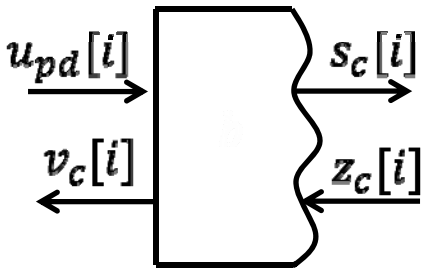
- Desired reference trajectory, y_d
  - Bounded and twice differentiable
- H_{mp} is passive (input-output mapping $f_p \mapsto s$)

Wave Variables



$$v_{cd}(t) = \frac{1}{\sqrt{2b}}(bs(t) - z(t))$$

$$u_p(t) = \frac{1}{\sqrt{2b}}(bs(t) + z(t))$$



$$v_c[i] = \frac{1}{\sqrt{2b}}(bs_c[i] - z_c[i])$$

$$u_{pd}[i] = \frac{1}{\sqrt{2b}}(bs_c[i] + z_c[i])$$

Variable Passive Sampler and Variable Passive Hold

- Passive Sample-and-Hold Operation

$$\int_0^{t_N} (u_p{}^T(t)u_p(t) - v_{cd}{}^T(t)v_{cd}(t))dt \geq \sum_{i=0}^{N-1} T_i(u_p{}^T[i]u_p[i] - v_{cd}{}^T[i]v_{cd}[i])$$

$$\int_0^{t_N} u_p{}^T(t)u_p(t)dt \geq \sum_{i=0}^{N-1} T_i u_p{}^T[i]u_p[i] \quad and \quad \int_0^{t_N} v_{cd}{}^T(t)v_{cd}(t)dt \leq \sum_{i=0}^{N-1} T_i v_{cd}{}^T[i]v_{cd}[i]$$

VPS

$$u_{p_j}[i] = \frac{1}{\sqrt{T_{i-1}T_i}} \int_{t_{i-1}}^{t_i} u_{p_j}(t)dt \quad \forall j \in \{1, \dots, m\}$$

VPH

$$v_{cd_j}(t) = v_{cd_j}[i] \quad t \in (t_i, t_{i+1})$$

Networked Controller

- Event-based proportional control

  $z_c[i] = K_c f_c$

  $where\ f_c = (s_c[i] - \Lambda r_c[i])\ and K_c = \mathrm{diag}\{K_{c1}, \dots, K_{cm}\}$

- Strictly-input passive system, $f_c \mapsto z_c$

$$\sum_{i=0}^{N-1} T_i z_c^{\mathrm{T}}[i] f_c[i] \geq \delta \sum_{i=0}^{N-1} T_i f_c^{\mathrm{T}}[i] f_c[i] - \beta_2$$

Sampling Policy

- Adaptive Sampling Mechanism
  - An adaptation of the self-triggered mechanism (Mazo and Tabuada)
  - Determines the next sampling interval, $T_i = t_{i+1} - t_i$
  - Based on a function of the tracking error.

$$\dot{e}(t) = -\Lambda e(t) + s(t)$$

$$s(t) = s(t_i) \qquad t \in [t_i, t_{i+1})$$

- Output Function: $V(e) = e^T P e$
- Performance Function: $S(t) = V\big(e(t_i)\big) exp^{-\rho(t-t_i)}$
- Continuous time sampling policy

$$h_c(t_i, e(t), e(t_i)) = V\big(e(t)\big) - V\big(e(t_i)\big) exp^{-\rho(t-t_i)} \leq 0$$

- Discrete-Time Equivalent
  - Given Δ, T_{min} and T_{max} such that $N_{min} := \left\lfloor \frac{T_{min}}{\Delta} \right\rfloor \qquad N_{max} := \left\lfloor \frac{T_{max}}{\Delta} \right\rfloor$

$$h_d(e(t_i), n) = h_c(t_i, e(t), e(t_i))$$

Sampling Policy

- Sampling Interval

$$T_i = \Gamma_d(e(t_i)) := \max\{T_{min}, n_i\Delta\}$$

$$where\ n_i := \max\{n \leq N_{max} | h_d(e(t_i), c) \leq 0, \quad c = 0, \ldots, n\}$$

Results

Theorem 1

- Given that the components of the PBASC architecture satisfy the passivity constraints.
- Assuming the handling of network messages is such that
  - No duplicate packet is processed
  - Null packets are processed in the case of packet loss and if the buffer is empty
- The input-output mapping $r_c \mapsto s$ is strictly-output passive

$$\int_0^{t_N} s^T r_c \, dt \geq \frac{b}{\sigma_{max}} \int_0^{t_N} s^T s \, dt - \frac{\beta_1}{\sigma_{max}}$$

Results

Theorem 2

- Consider the proposed PBASC architecture with the composite system, H_{mp} and the networked controller, H_c and based on the previous passivity theorem
  - Assuming H_{mp} is zero state observable
- The equilibrium, s_a is asymptotically stable.
- Hence

$$\lim_{t\to\infty} e_a(t) = \lim_{t\to\infty} y_p(t) - y_{da}(t) = 0$$

CASE STUDY

- Trajectory tracking of robotic manipulator over a wireless network
  - Crustcrawler Smart Arm
    - Four degrees-of-freedom
- Dynamics of the manipulator

$$M(y_p)\ddot{y}_p + C(y_p, \dot{y}_p)\dot{y}_p + g(y_p) = \tau$$

- Composite system, H_{mp}

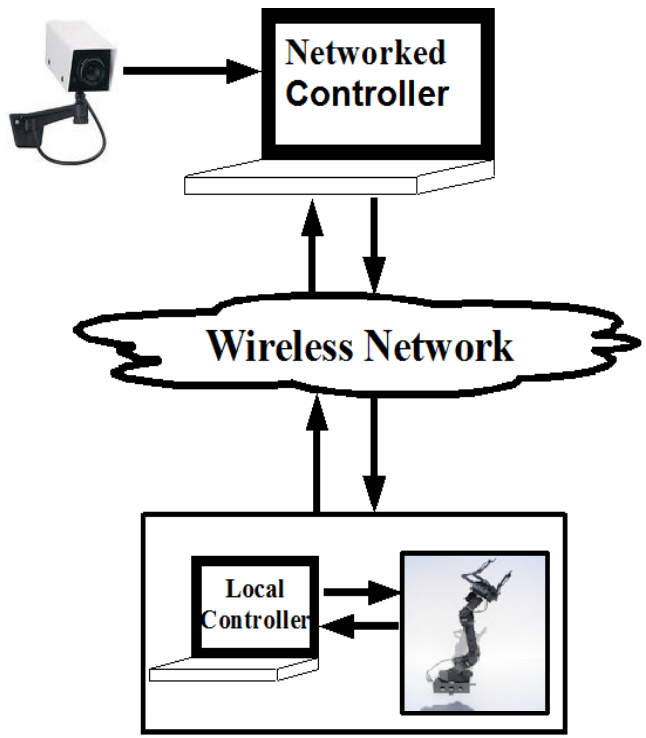
$$\tau = M(y_p)\ddot{y}_r + C(y_p, \dot{y}_p)\dot{y}_r + g(y_p) + f_p$$

$$y_r = y_d - \Lambda \int e\, dt$$

$$\dot{y}_r = \dot{y}_d - \Lambda e$$

$$s = \dot{y}_p - \dot{y}_r = \dot{e} + \Lambda e$$

$$\boldsymbol{M\dot{s} + C\dot{s} = f_p}$$



CASE STUDY

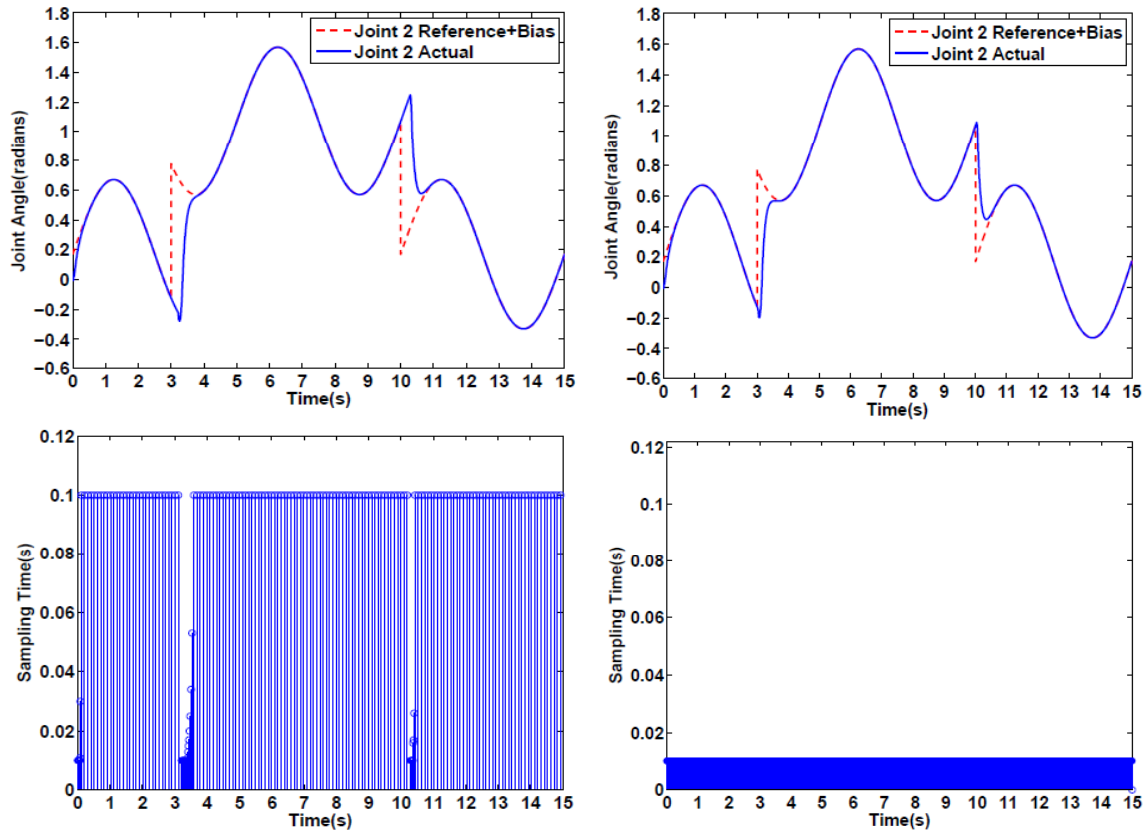
- Matlab/Simulink
  - Composite System
  - Networked Controller
- TrueTime
  - Model of the wireless network

Parameter	Value
T_{min}	$0.01s$
T_{max}	$0.1s$
Δ	$0.001s$
K_c	I
b	1
Λ	$10I$
P	$0.5I$
$[m_1, m_2, m_3, m_4]$	$[0.362, 0.401, 0.059, 0.177]kg$

- Evaluation
  - Comparison of PBASC approach and fixed sampling interval (FSP)
  - Presence of network uncertainties (packet loss and time-varying delay)

Comparison of FSP and PBASC

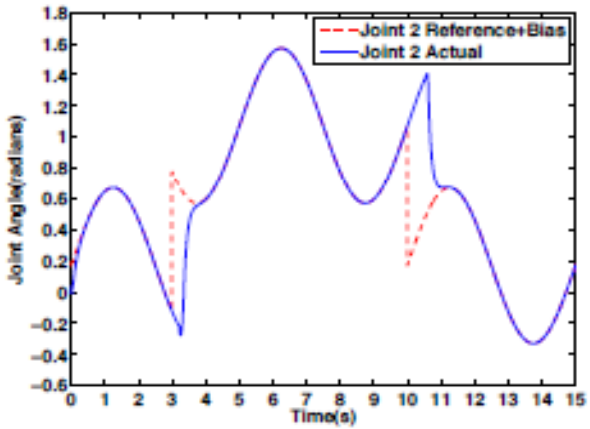
- Joint 2
  - Rest position (0.17 rad)
  - Reference
    - $0.5\sin(\frac{2\pi}{5})$
  - Bias (0.9 rad)

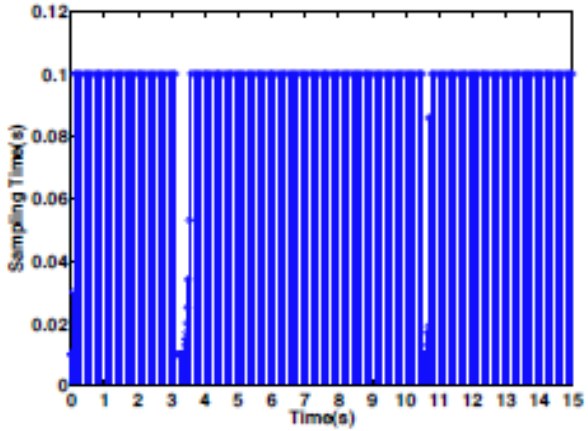


PBASC **FSP**

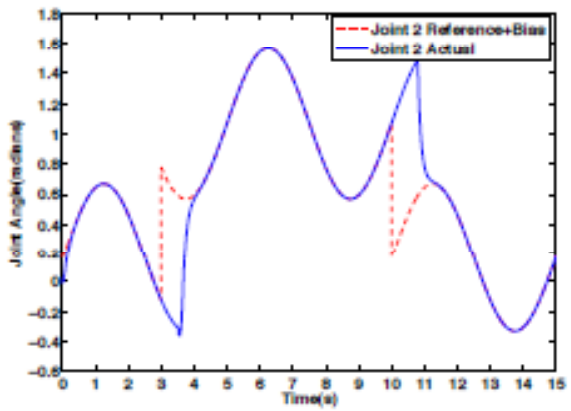
Presence of Network Uncertainties

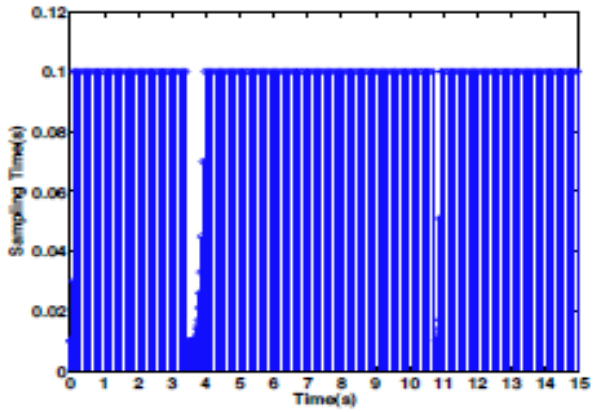
- Time-varying delay





- Packet Loss





Conclusion

- Introduced a passivity based adaptive sampling architecture
  - Trajectory tracking
- Under appropriate assumptions
  - Architecture is shown to be strictly output passive
  - Trajectory tracking is achieved
- Approach
  - Demonstrates an efficient use of networked and computational resources.
  - Stability is maintained and performance degrades gracefully in the presence of network time varying delay and packet loss.

THANK YOU