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Criticality Safety Analysis Uncertainty Evaluations Using Bayesian Hierarchical Procedures

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Overview

- Observations: Sources of Uncertainty
- Observation: Hierarchy of Uncertainties
- Bayesian Hierarchical Procedures
- Criticality Analysis Validation
- Criticality Analysis Application Case
- BUC: Depletion Calculation Validation → Cordoba meeting

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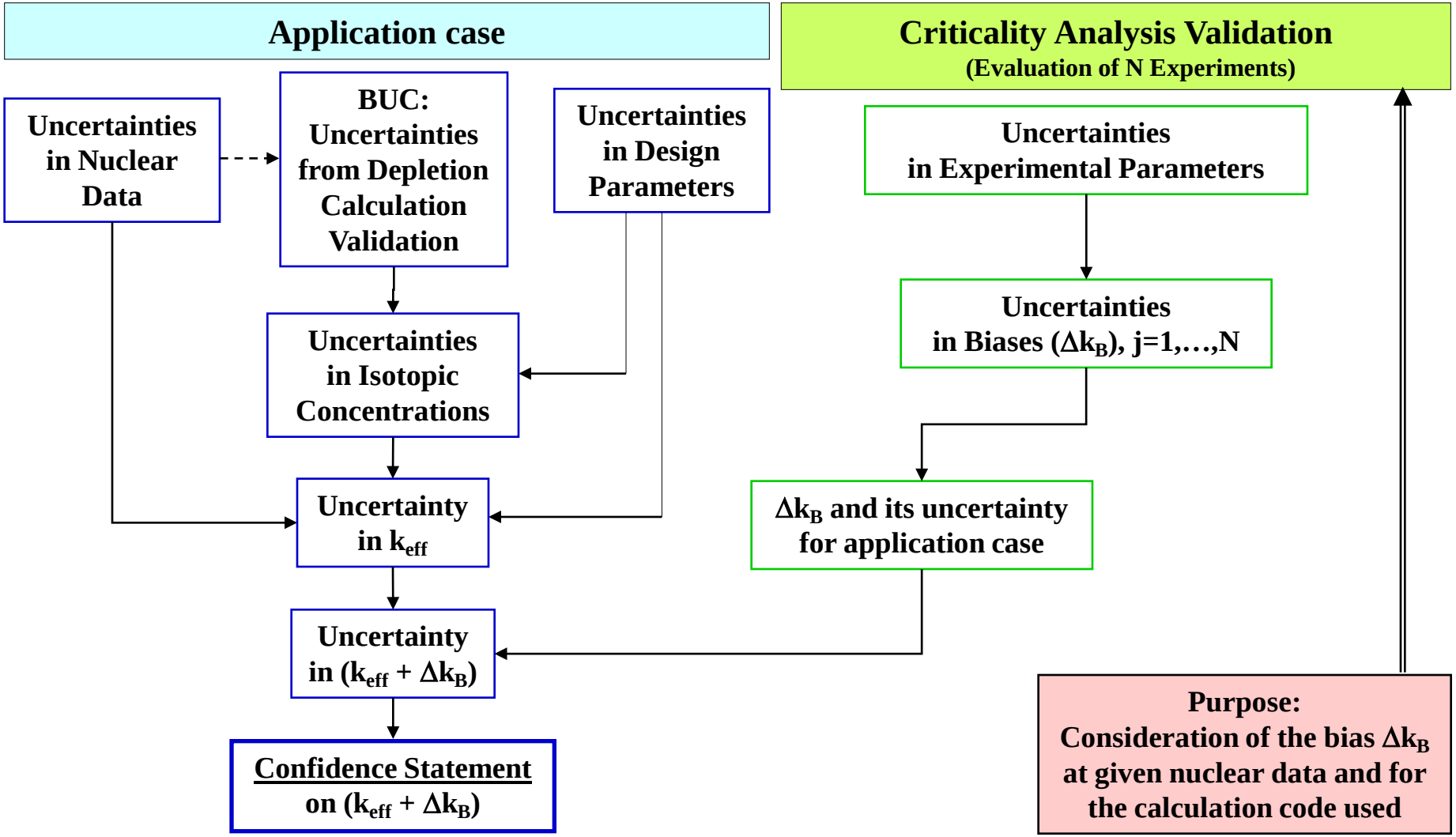
Observations: Sources of Uncertainty



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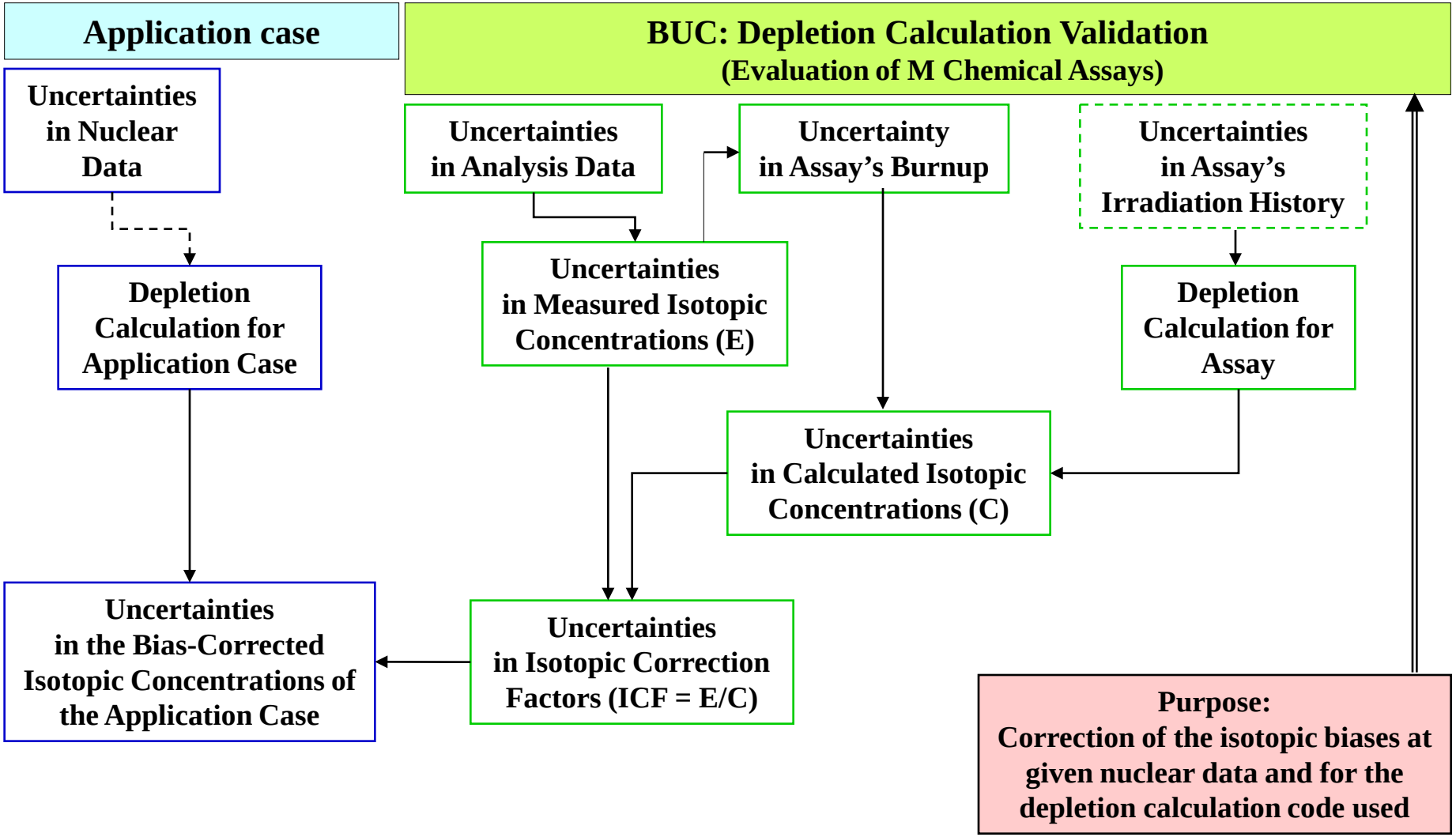
Observations: Sources of Uncertainty



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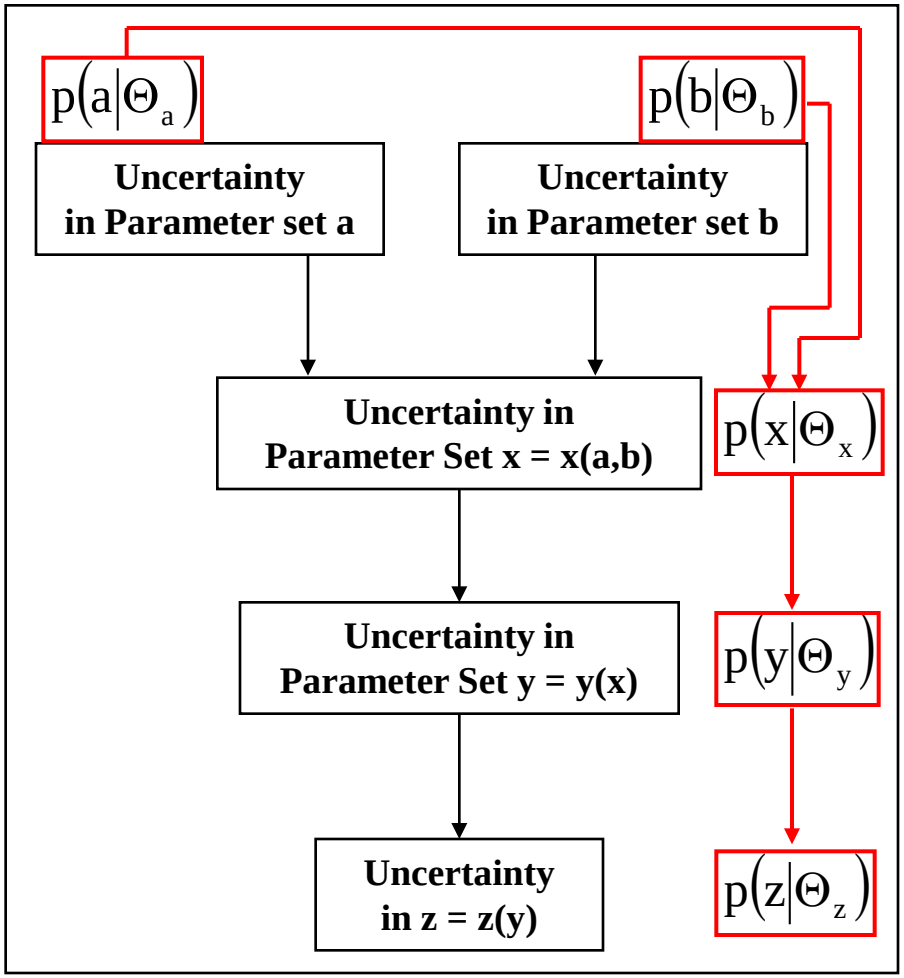
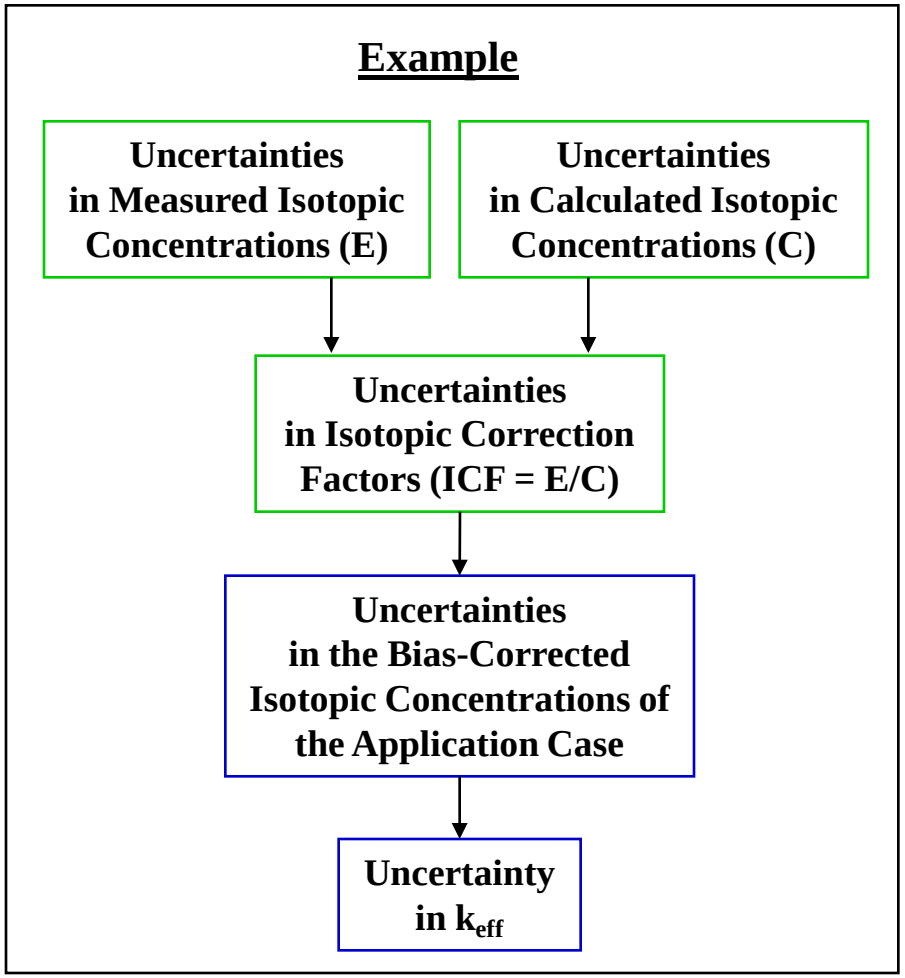
Observation: Hierarchy of Uncertainties





Most powerful tool of bearing the uncertainties from one level to the next one:
→ Use of Bayesian hierarchical procedures

← NCSD 2009 Topical Meeting, Sept. 13-17, 2009 Paper #33 (Neuber, Hoefer)

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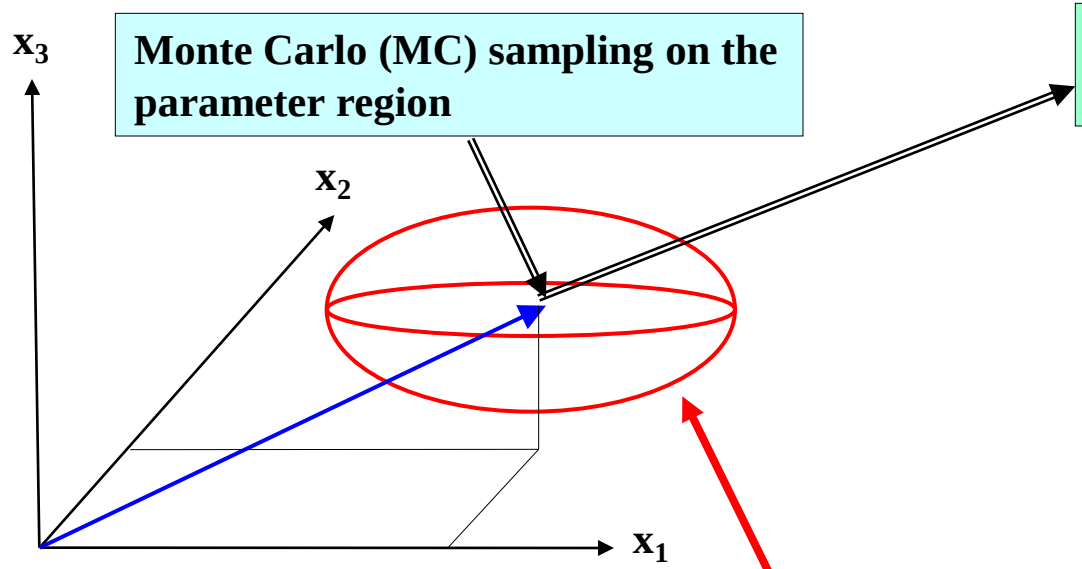
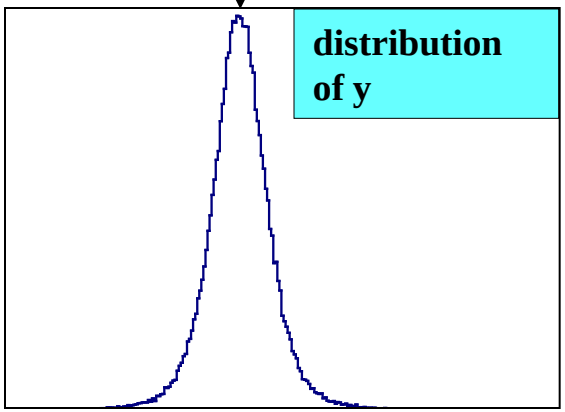
Monte Carlo Sampling at given level



Monte Carlo (MC) sampling on the parameter region

Sets of MC sampled parameter values
$(x_s)_i = (x_{s1}, x_{s2}, x_{s3}, \ldots)_i, \; i = 1,\ldots,\kappa$

Set of MC sampled parameter values
$(y_s)_i = y((x_s)_i), \; i = 1,\ldots,\kappa$



- **MC sampling on a parameter region requires a joint probability density function (pdf) $p(x|\Theta)$ of the parameters x**
- **Problem: pdf usually unknown**
- **Necessary: *pdf model derived from empirical data***

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Monte Carlo Sampling at given level

→Generate MC samples x_s under the condition of empirical data X:

Posterior predictive

$$p(x_s|X) = \int p(x_s|\Theta)\,p(\Theta|X)\,d\Theta$$

n x m *data matrix* of n independent identically distributed (iid) m-variate data $x_i = (x_{i1}, x_{i2}, \ldots, x_{im})$

$$\begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1,m-1} & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2,m-1} & x_{2m} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{n-1,1} & x_{n-1,2} & \cdots & x_{n-1,m-1} & x_{n-1,m} \\ x_{n,1} & x_{n,2} & \cdots & x_{n,m-1} & x_{n,m} \end{pmatrix}$$

Θ → probability distribution model

e.g. normal distribution: $\Theta = (\mu, \Sigma)$

parameter Θ unknown

→MC sampling on Θ under the condition of the data X

$$p(\Theta|X) \propto p(X|\Theta)\,p(\Theta)$$

posterior know-ledge about Θ

Likelihood of X under Θ

prior knowledge about Θ

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Simple example for Monte Carlo Sampling at given level

Empirical data $x_1, x_2, x_3, \ldots$

$$p(x \mid \mu, \sigma^2) = (2\pi\sigma^2)^{-1/2} \exp[-(x-\mu)^2 / 2\sigma^2] = N(\mu, \sigma^2)$$

$$\bar{x} = \sum_{i=1}^{n} x_i / n$$

$$\hat{\sigma}^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2 / (n-1)$$

$$\mu \sim N(\bar{x}, \sigma^2 / n)$$

MC

$$\sigma^2 \sim \mathrm{Inv}-\chi^2(n-1; \hat{\sigma}^{-2})$$

MC

MC

$$p(x \mid \mu, \sigma^2) = N(\mu, \sigma^2)$$

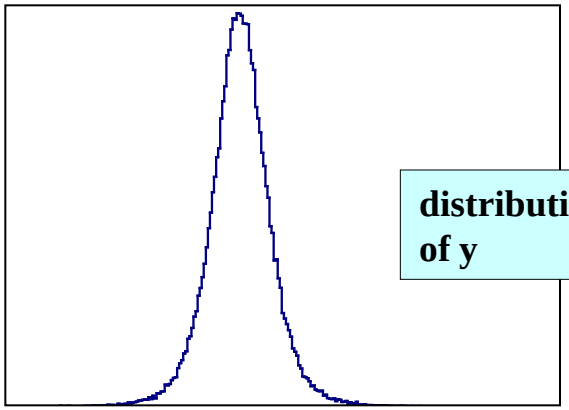
MC

$$x_{MC}$$

$$y_{MC} = y(x_{MC})$$

next MC sample

(sampling loop)



distribution of y

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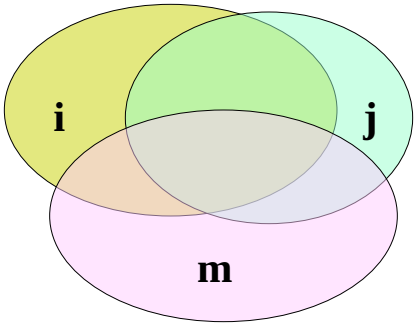
Criticality Analysis Validation

In many cases: "mutually dependent experiments"

Space of experimental parameters x of <u>all</u> the experiments



Monte Carlo sampling on entire x space

↓

For each sampled vector x_{MC} calculation of the k_{eff} values (k_1, k_2, …,k_N) for all the N experiments

↓

Bias vector (Δk_{B1}, Δk_{B2}, …,Δk_{Bn}) for all the N experiments

↓

Bayesian linear regression with this bias vector using adequate explanatory variables

↓

MC sample of the bias Δk_B for the application case

↓

Add to k_{calc} of application case: $k_{calc}+\Delta k_B$

Monte Carlo sampling for application case → k_{calc}

Empirical distribution of ($k_{calc}+\Delta k_B$)

MOCADATA code package

NCSD 2009 Topical Meeting, Sept. 13-17, 2009
Paper #33 (Neuber, Hoefer)

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Criticality Analysis Application Case

Monte Carlo sampling on system parameter x and involved nuclear data ξ

↓

Calculation of the k_{eff} value for the sampled vector $(x,\xi) \rightarrow k_{calc}$

Add sampled Δk_B of application case to k_{calc} : $k_{calc} + \Delta k_B$

Monte Carlo sampling on bias Δk_B

Empirical distribution of $(k_{calc} + \Delta k_B)$

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Criticality Analysis Application Case

Uncertainty of nuclear data: Monte Carlo sampling on involved nuclear data ξ

$$p\left(\boldsymbol{\Theta}(E_n)\middle|\,\mathbf{M}\right) \propto p\left(\mathbf{M}\middle|\,\boldsymbol{\Theta}(E_n)\right) p\left(\boldsymbol{\Theta}(E_n)\right)$$

Matrix of empirically based basic nuclear data

Neutron energy

i+1

i-th MC sample on Θ

$$p\left(\xi_{BD}^{(S)}(E_n)\middle|\,\boldsymbol{\Theta}(E_n)\right)$$

Basic data

i-th MC sample on ξ_{BD}

Basic data evaluation codes

Point data (continuous x-sections)

Application case

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Criticality Analysis Application Case

Uncertainty of Nuclear Data: Use of TSUNAMI

Perform n TSUNAMI calculations using different starting random numbers

Results $(\sigma_{ND})_1, \ldots, (\sigma_{ND})_1$

$$\bar{\sigma}_{ND} = \frac{1}{n}\sum_{i=1}^{n}(\sigma_{ND})_i$$

$$\hat{\tau}^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left((\sigma_{ND})_i - \bar{\sigma}_{ND}\right)^2$$

$$\mathrm{Inv}-\chi^2(n-1;\hat{\tau}^{-2}) \rightarrow \tau^2_{MC}$$

$$N(\bar{\sigma}_{ND}, \tau^2_{MC}/n) \rightarrow E[\sigma_{ND}]_{MC}$$

$$N(E[\sigma_{ND}]_{MC}, \tau^2_{MC}) \rightarrow (\sigma_{ND})_{MC}$$

$$N\left(0, (\sigma_{ND})^2_{MC}\right) \rightarrow \Delta k^{MC}_{ND}$$

$$\text{Add to } k^{MC}_{calc} \text{ and } \Delta k^{MC}_{B} \rightarrow [k_{calc} + \Delta k_{ND} + \Delta k_B]^{MC}$$

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BUC: Depletion Calculation Validation

Several steps required:

- **Measured concentrations**
- **Calculated concentrations**
- **Isotopic Correction Factors (ICF)**
- **Reduction to BUC isotopes ← application case**
- **Incomplete data sets of BUC-ICFs → Solution of missing data problem**
- **Sampling on BUC-ICFs**

PHYSOR 2008, Paper #525 (Neuber, Hoefer)

Upcoming CSN/IAEA BUC Workshop, Cordoba 2009

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